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CUMULATIVE THERMAL EFFECTS IN A

MULTIBURST SCENARIO

THESIS

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
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Master of Science in Nuclear Engineering

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Preface

I would like to thank my advisor, Maj. John F. Prince, Associate Professor of Physics, Air Force Institute of Technology, for proposing this thesis topic and for his guidance during the thesis effort. I would also like to thank Phyllis Reynolds for her help in preparing this manuscript. Finally, a special thanks to my husband Dave for his love, support, and patience during this study.

— Barbara A. Hall

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List of Symbols

a	$a = c_p \rho d$ (J/m^2-°K)
CF	correction factor: sin (ψ)
CT	fraction of thermal energy emitted from burst
ΔCT_j	fraction of thermal energy emitted during time step j
c_p	specific heat capacity (J/kg-°K)
d	missile skin thickness (m)
DGZ	designated ground zero
drd	down-range-distance
$\dot{F}$	rate of heating (J/m^2-s)
$F_{absorbed}$	amount of incident thermal radiation that is absorbed by the missile skin (J/m^2)
$\dot{F}_{absorbed}$	time derivative of $F_{absorbed}$ (J/m^2-s)
$F_{convection}$	amount of absorbed radiation that is lost to convective cooling by air (J/m^2)
$\dot{F}_{convection}$	time derivative of $F_{convection}$ (J/m^2-s)
F_{heat}	total amount of energy available to heat the missile skin at its surface (J/m^2)
$\dot{F}_{heat}$	time derivative of F_{heat} (J/m^2-s)
$F_{incident}$	radiant exposure from a burst (J/m^2)
$F_{radiation}$	amount of absorbed radiation that is re-radiated as IR black-body radiation to the environment (J/m^2)

$\dot{F}_{radiation}$	time derivative of $F_{radiation}$ $(J/m^2\text{-}s)$
GR	ground range (m)
h	heat transfer coefficient $(J/m^2\text{-}s\text{-}°K)$
Hfb	height of fireball (m)
I_{ss}	sure-safe intensity (°K)
I_{sk}	sure-kill intensity (°K)
j	time step
k	thermal conductivity (J/m-s-°K)
k	burst #
lt	launch time (s)
MaxT	maximum temperature (°K)
$P_d(I)$	probability of damage due to intensity I
P_d	total probability of damage for N cells
P_{d_i}(MaxT)	probability of damage due to the maximum temperature calculated for cell i
P_s	probability of survival
ΔQ_j	thermal fluence incident on the missile skin during time step j (J/m^2)
SR	slant range (m)
t	time after a burst (s)
t′	normalized time t/t_{max}
tb_1	time of first burst (s)
t_{diff}	thermal diffusion time (s)
t_m	time after missile launch (s)
t_{max}	time after second thermal maximum (s)
t_0	$t_0 = tb_1 - lt$

$T(x,t)$	skin temperature (°K)
T_1, T_2	skin temperature at beginning and end of time step j (°K)
$T_{air}(t)$	ambient air temperature (°K)
tf	thermal fraction
W	yield in megatons
Y	yield in kilotons
x_m	point on missile skin (m)
α	absorptivity
α'	parameter for $P_d(I)$
β'	parameter for $P_d(I)$
K	thermal diffusivity (m^2/s)
ψ	angle between slant range and missile skin
ρ	density (kg/m^3)
ρ_i	distance to centroid of cell i
ϕ	angle through dust cloud
θ	missile flight path angle
θ_i	angular location of centroid of cell i
σ	Stefan-Boltzmann constant
σ	statistical variation
τ	transmittance

Abstract

In current survivability studies, the nuclear bursts are treated as independent events. Using this method, the effect of thermal radiation from one fireball at a time is studied. This treatment does not consider the cumulative effects of receiving thermal radiation from more than one fireball at a time. The purpose of this

This thesis sought ~~was~~ to develop a computer program to model the cumulative effects of nuclear fireball thermal radiation and compare these results to those from the noncumulative case.

The scenario studied was the Peacekeeper Dense Pack missile system. The missile field was subjected to a walk attack of 2 MT weapons every two seconds. a ~~The~~ aiming error of the incoming RV was modeled using a 10-cell ~~cir-~~ CEP cular error probable (CEP) area around the designated ground zero, and the probability of damage due to an RV was calculated using a cumulative log-normal distribution function.

In order to model the missile skin temperature rise of the missile skin, an energy balance was made over a unit area of skin surface and then solved using the thin skin approximation and finite differences. The resulting equation gave an expression for the skin temperature at a time t

cont

after the first burst detonated. The maximum temperature reached was ~~then~~ used to calculate the probability of damage to the missile skin.

To model cumulative thermal effects, the amount of thermal radiation emitted from each burst was added together to calculate ~~the~~ skin temperature. This method resulted in temperatures ~~that were~~ significantly higher than were the temperatures calculated for each burst independently. ~~Consequently,~~ cumulative thermal effects thus proved to have a greater region of no survival than noncumulative thermal effects and also blast effects.

CUMULATIVE THERMAL EFFECTS IN A MULTIBURST SCENARIO

I. Introduction

Background

A nuclear weapon releases a large portion of its energy in the form of electromagnetic radiation. This radiation, which is emitted within a microsecond after the blast, is called primary radiation. Since the primary radiation is emitted at tens of millions of degrees, it is in the soft X-ray region of the electromagnetic radiation spectrum. Therefore, for a low altitude or surface burst, the radiation is almost immediately absorbed by the atmosphere. As a result, the air is heated and forms a fireball that in turn emits thermal radiation in the ultraviolet to infrared regions of the electromagnetic radiation spectrum. This secondary thermal radiation travels a long distance from the burst in a short time. The fraction of the bomb yield that is emitted as effective thermal radiation (primary plus secondary) depends on the height of the burst, the total yield, and other weapon characteristics.

As the thermal radiation travels through the air, it diverges and is attenuated by absorption and scattering processes with air molecules and other particles. These processes depend on the wavelength of the radiation and atmospheric conditions that vary with altitude and the size and density of the interacting particles. Thus, the amount of thermal radiation incident on a target is determined by the total energy yield, the height of the burst, the distance from the target to the burst, and the changing characteristics of the atmosphere.

At the target, a fraction of the incident thermal radiation will be absorbed. For a given surface material, only a small amount of the absorbed energy will be dissipated away from the surface by conduction, convection, or re-radiation. Therefore, the absorbed energy is contained in a shallow depth of the target skin, resulting in high temperatures that could damage the skin material. The purpose of a survivability study is to determine what this skin temperature will be in order to predict the target's probability of survival. If the skin temperature is above the sure-kill temperature of the skin material, then the probability of survival will be zero. If the temperature is below the sure-safe temperature, the probability of survival will be one.

In current multiburst scenarios, the thermal effect due to each burst is treated as an independent event, meaning that the temperature rise of the target skin is considered to be the result of thermal radiation emitted from only one fireball. For a series of bursts, the temperature rise would be monitored individually, resulting in a probability of survival for each burst. The probabilities would then be combined to arrive at a final probability of survival for the target. This method of treating bursts as independent events will be called the noncumulative case. The cumulative case is the case of considering the temperature rise of the target skin to be the result of receiving thermal radiation from more than one fireball at the same time.

Problem and Scope

The purpose of this thesis project was to develop a computer program to calculate the skin temperature rise of a missile subjected to cumulative thermal effects from a series of bursts occurring during the missile's flight. The probability of survival for this case was then compared to the probability calculated for the noncumulative case to demonstrate the need for considering cumulative effects. Furthermore, since blast effects are generally considered more lethal than thermal effects in the noncumulative case, the cumulative results were also compared to results from noncumulative blast effects to determine if thermal radiation becomes the kill mechanism. During this research, no effort was made to vary the threat conditions or to optimize the probability of survival for a missile. Instead, the intent was to show that treating each burst as an independent event gives results that severely overestimate a missile's probability of survival.

The burst-target missile system studied was the Peacekeeper close-spaced basing (CSB) system, more commonly known as Dense Pack. Although this system is no longer being considered for national defense, it is useful for showing the differences between the cumulative and noncumulative cases. The configuration of the missile field is shown in figure 1. The field is subjected to a walk attack starting at silo #1 and proceeding every two seconds to silos #2, #3, #4, and so on. The basic scenario studied was that of a missile launching at the same time as silo #1 is hit.

Assumptions and Limitations

The following assumptions or limitations were made to simplify the analysis:

1. Cratering and blast effects produced by the burst are not modeled.

2. The fireball is considered to be an isotropic point source of thermal radiation. Furthermore, the point source is assumed to be centered at the top of the rising dust cloud created by the fireball.

3. The amount of scatter and absorption of thermal radiation in the air is quantified by using the transmittance τ, which is the fraction of direct and scattered radiation transmitted from burst to target. An expression

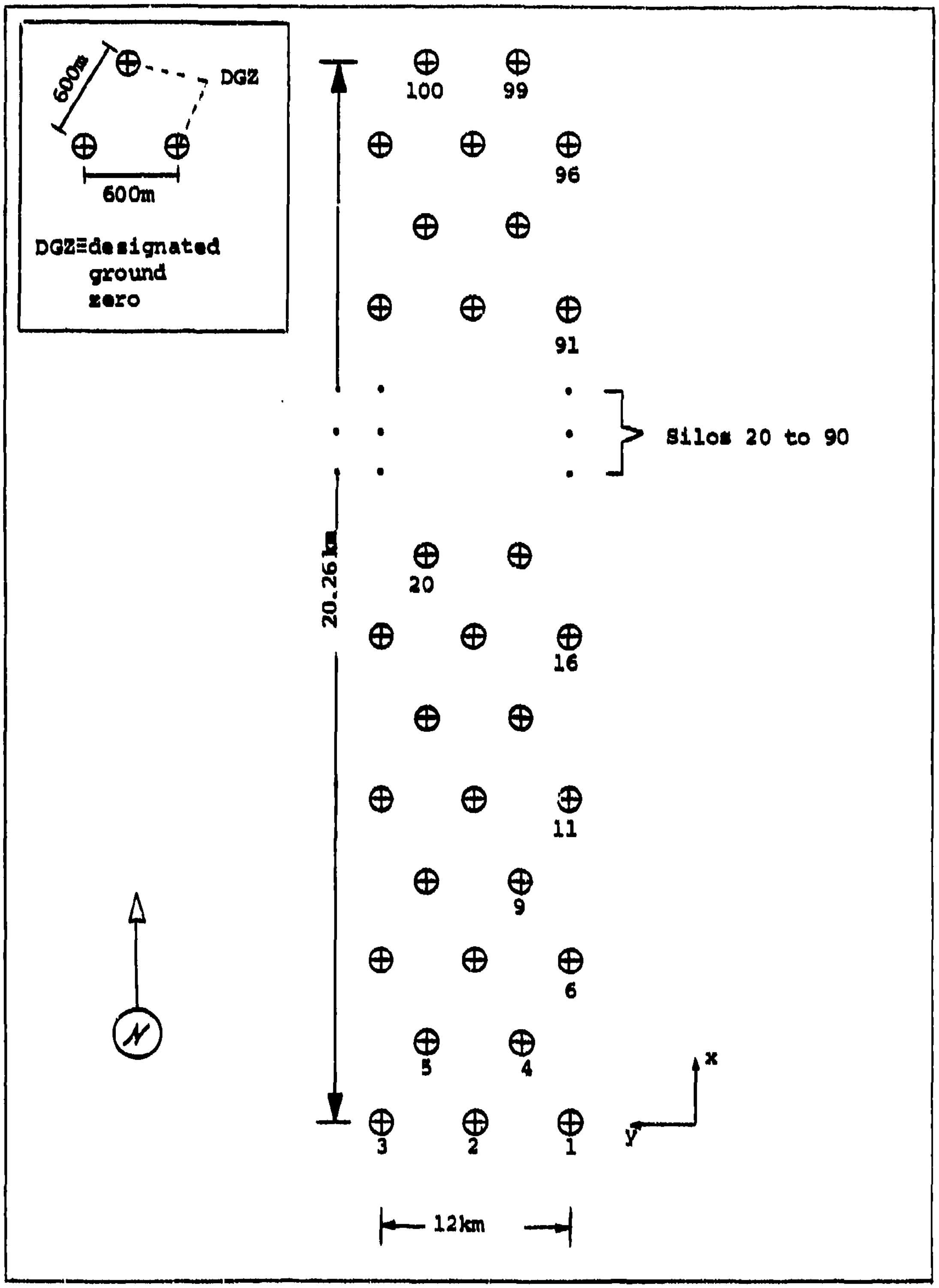


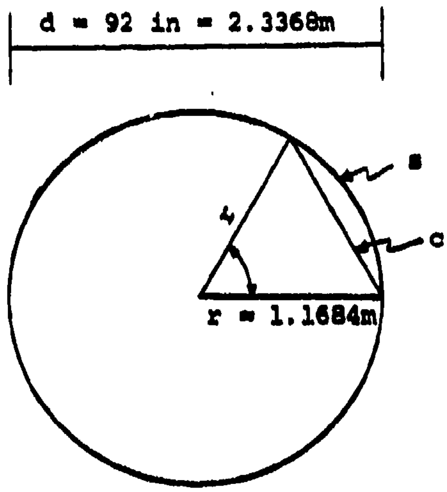
Fig. 1. Dense Pack Missile Field

for τ as a function of slant range will be developed from data presented for a cloudless atmosphere with a visibility of ~20 km (Glasstone and Dolan, 1977:318). This expression for τ does not account for the decrease in transmitted radiation due to the dust that has been lifted into the air.

4. The thermal conductivity and specific heat of the missile skin material are assumed to be independent of temperature and therefore constant during the heating of the skin. This is a reasonable assumption for temperatures between 619 °K and 809 °K [Touloukian et al., 1970: 1 (Vol 1) and 1 (Vol 4)], the sure-safe and sure-kill intensities chosen for an aluminum missile skin. Although the assumption is not true for temperatures above the melting point of the missile skin, the change in material properties is not important compared to the fact that melting constitutes a 100% failure of the missile.

5. The missile skin surface is modeled as a flat plate rather than as its actual shape of a cylinder. This assumption is reasonable for an energy balance made over a square meter of skin surface. For a Peacekeeper missile with a diameter of 2.3 m (Young, 1984:168), the difference between a flat surface and a curved surface is ~3%, as shown in figure 2.

Also, the slant range from burst to missile is not calculated for a specific point along the missile's length.



$$s = r\theta \qquad c = 2r\sin\frac{\theta}{2}$$

$$\%\ \text{difference} = \frac{A_{\substack{\text{curved}\\ \text{surface}}} - A_{\substack{\text{flat}\\ \text{surface}}}}{A_{\substack{\text{curved}\\ \text{surface}}}}$$

For a square meter of flat surface:

$c = 1\text{m}$
$\theta = .8844$
$s = 1.033\text{m}$

Missile cross-section

% difference = 3.2%

Fig. 2. Comparing a Flat and Curved Surface

Again, for a Peacekeeper missile with a length of ~21 m (Young, 1984:168), the difference in slant ranges calculated from either end of the missile would not be significant for the distances involved.

6. The missile's position and velocity at any time are given in figure 3. The missile has no velocity component in the y direction and therefore flies straight north, as indicated in figure 1.

7. The walk attack is assumed to occur with perfect timing and at a perfect height of burst of 0 m (i.e., a surface burst). Thus, the only aiming error considered is the RV's horizontal distance from designated ground zero,

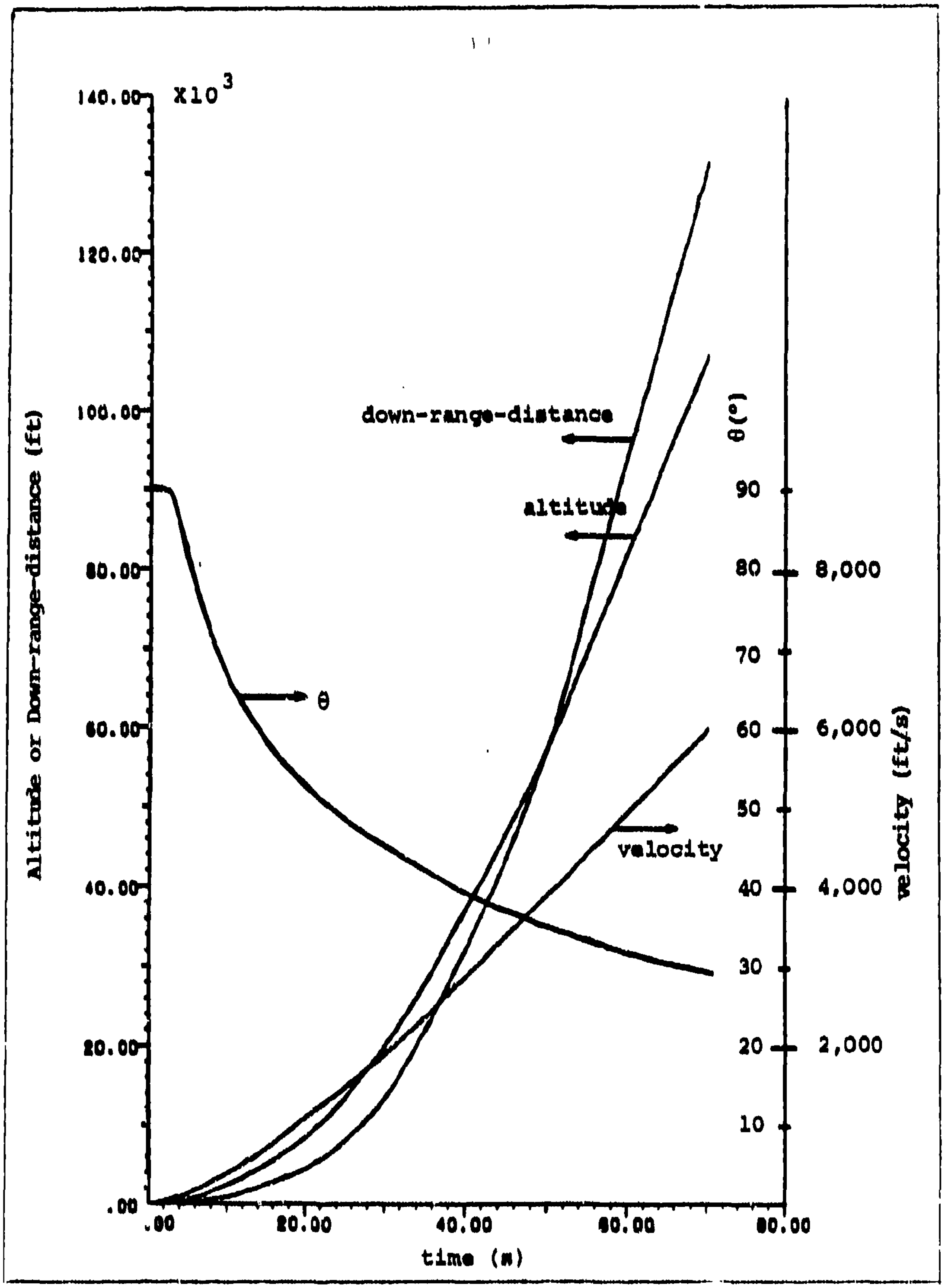


Fig. 3. Missile Characteristics

where designated ground zero is the location of the targeted silo.

8. The probability of damage is a function of intensity rather than range and is described using a cumulative log-normal distribution function.

9. The analysis considers only bursts that occur on or after a missile's launch time. This assumption simplifies the computer simulation of the problem.

Approach

An expression for the rate of heating at the surface of the missile skin was derived from an energy balance over a square meter of the flat surface. This balance included absorption, convection, and re-radiation, but it was later determined that re-radiation was not significant for the temperatures of interest. The energy balance was then solved for the missile skin temperature using a simplification called the thin skin approximation. The calculation of the maximum skin temperature for a series of bursts involved an iterative process that modeled the cumulative effect of more than one burst by combining the amount of thermal radiation emitted by each burst at a particular time. The maximum skin temperature was then used to calculate the probability of survival for the missile.

Presentation

The derivation of the equation for the missile skin temperature, a discussion of how the probabilities were calculated, and an explanation of how this information was used to determine the probability of survival in both the cumulative and noncumulative cases is presented in Chapter II. Chapter III summarizes the conditions and parameters used in this study and explains the reasoning behind the choice of the parameters. The results of the study are discussed in Chapter IV and conclusions and recommendations are presented in Chapter V.

II. Theory

This chapter contains the theory used to determine the probability of survival of a missile subjected to thermal radiation in both the cumulative and noncumulative cases. First, the derivation of the equation for the maximum skin temperature is presented, followed by an explanation of how this equation is used for a single burst. Next, the probability of damage function and aiming error are introduced. This information is then used to explain how the probability of survival is calculated for the noncumulative and cumulative case.

Derivation of the Equation for Maximum Skin Temperature

The missile's probability of survival for the thermal threat is calculated using the maximum missile skin temperature reached during a burst scenario. An equation for this temperature is derived using an energy balance over a unit area of missile skin surface and a simplification known as the thin skin approximation. The following presentation of the derivation is taken from McKee (McKee, 1984:1-8).

The cylindrical shape of the missile is not considered for this derivation. Instead, the missile skin is modeled as a finite slab of thickness d, with no heat lost from the back wall of the skin ($x = d$) and no heating from air friction. The energy balance for a unit area of missile skin surface ($x = 0$) in terms of fluence is:

$$F_{heat} = F_{absorbed} - F_{convection} - F_{radiation} \tag{2.1}$$

where

F_{heat} = Total amount of energy/m^2 available to heat the missile skin at $x = 0$

$F_{absorbed}$ = Amount of incident thermal radiation that is absorbed by the missile skin

$F_{convection}$ = Amount of absorbed radiation that is lost to convective cooling by air

$F_{radiation}$ = Amount of absorbed radiation that is re-radiated as IR black-body radiation to the environment

Since this equation must hold for all times, the time derivative may be obtained to yield the following differential equation:

$$\dot{F}_{heat} = \dot{F}_{absorbed} - \dot{F}_{convection} - \dot{F}_{radiation} \tag{2.2}$$

where the dots indicate the time derivative or a rate.

The total amount of energy available for heating, F_{heat}, will be conducted from the skin surface through the skin thickness. If the flow of heat is assumed to be one-dimensional and the properties of the medium are constant, then the conduction process is described by the heat transfer equation (Holman, 1976:102):

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{K}\frac{\partial T(x,t)}{\partial t} \tag{2.3}$$

where

K = thermal diffusivity (m^2/sec) = $\frac{k}{c_p \rho}$

k = thermal conductivity (J/m-s-°K)

c_p = specific heat capacity (J/kg-°K)

ρ = density (kg/m^3)

$T(x,t)$ = skin temperature (°K) at depth x and time t

This equation can be used to estimate the thermal diffusion time t_{diff} required for heat to be conducted through the skin thickness d (see Appendix A). The result is:

$$t_{diff} = d^2/K \tag{2.4}$$

If t_{diff} is much less than the time scale over which the thermal pulse occurs, the missile skin will essentially be at a uniform temperature throughout x at any given time t. The thermal pulse time scale is t_{max}, the time of the second thermal maximum. For air bursts below 4572 m, the expression for t_{max} is (Glasstone and Dolan, 1977:310):

$$t_{max} = 0.0417\ Y^{.44} \tag{2.5}$$

where Y is the yield in kilotons. This expression will also be used for surface bursts. Thus, for uniform heating to occur:

$$t_{diff} << t_{max} \quad \text{or} \quad d << (K \cdot t_{max})^{.5} \tag{2.6}$$

If d fulfills the above requirement, then the thin skin approximation of uniform temperature throughout the skin thickness is valid. As an example, t_{max} is ~1.18 sec for a 2 MT burst and the thermal diffusivity of aluminum is 1 cm^2/sec, so that $(K \cdot t_{max})^{.5}$ is ~1.0 cm. Therefore, a reasonable value for d would be ten times less than this value, or 0.1 cm.

An expression for $\dot{F}_{heat}$ in equation (2.2) can be derived from equation (2.3), the thin skin approximation, and Fourier's law of heat conduction (Holman, 1976:2):

$$\dot{F}(x,t) = -k \frac{\partial T(x,t)}{\partial x} \tag{2.7}$$

where $\dot{F}$ is the rate of heating. Differentiating (2.7) with respect to x and substituting into (2.3) yields:

$$-\frac{\partial \dot{F}(x,t)}{\partial x} = c_p \rho \frac{\partial T(x,t)}{\partial t} \tag{2.8}$$

If the thin skin approximation applies, then T(x,t) becomes a function of time only and the temperature at x = 0 equals the temperature at x = d equals T(t). The above equation can then be integrated over the skin thickness:

$$-\int_0^d \frac{\partial \dot{F}(x,t)}{\partial x}\, dx = \int_0^d c_p \rho \frac{\partial T(t)}{\partial t}\, dx \tag{2.9}$$

to obtain:

$$\dot{F}(0,t) - \dot{F}(d,t) = c_p \rho d \left(\frac{dT(t)}{dt}\right) \tag{2.10}$$

By the definition of $\dot{F}_{heat}$ and the assumption that no heat is lost at $x = d$, the final result is:

$$\dot{F}_{heat} = a \frac{dT(t)}{dt} \tag{2.11}$$

where $a = c_p \rho d$.

The amount of incident radiation absorbed is:

$$F_{absorbed} = \alpha F_{incident} \tag{2.12}$$

where

α = absorptivity of missile skin

$F_{incident}$ = radiant exposure from a burst (Glasstone and Dolan, 1977:316)

$$= CF \cdot \frac{tf \cdot y \cdot 4.186 X 10^{12} \cdot \tau}{4\pi (SR)^2} \; (J/m^2) \tag{2.13}$$

tf = thermal fraction or effective thermal partition: fraction of bomb yield appearing in the form of thermal radiation = .18 for a surface burst (Glasstone and Dolan, 1977:319)

y = bomb yield (kt)

τ = transmittance of atmosphere

SR = slant range from thermal radiation point source to missile (m)

CF = a correction factor: $\sin(\psi)$, where ψ = angle between missile skin surface and slant range vector

The energy absorption rate is:

$$\dot{F}_{absorbed} = \alpha \frac{dF_{incident}}{dt} \tag{2.14}$$

The convective cooling term can be written immediately as a rate as follows (Holman, 1976:12):

$$\dot{F}_{convection} = h[T(t) - T_{air}(t)] \tag{2.15}$$

where

h = local convective heat transfer coefficient (J/m^2-s-°K)

$T_{air}(t)$ = temperature of ambient air at missile altitude (°K)

The variable h depends on the velocity and temperature of the air flowing along the missile and is calculated for a specific point on the missile skin. Appendix E describes how h is calculated for a flat plate heated to a uniform temperature over its entire length.

The expression for the rate of black-body radiation is (Holman, 1976:13):

$$\dot{F}_{radiation} = \sigma[T(t)^4 - T_{air}(t)^4] \tag{2.16}$$

where σ is the Stefan-Boltzmann constant = 5.6696×10^{-8} J/m^2-s-°K^4. The radiation term will be insignificant for skin temperatures below 1000 °K, which is the case for the scenarios considered in this study. Thus, radiation is neglected in the analysis.

Equation (2.2) can now be written as:

$$a \frac{dT(t)}{dt} = \alpha \frac{dF_{incident}}{dt} - h[T(t) - T_{air}(t)] \tag{2.17}$$

This differential equation is solved using the method of finite differences. Choosing t_{max} as a finite time step, the following replacements are made:

$$\frac{dT(t)}{dt} \rightarrow \frac{T_2 - T_1}{t_{max}}$$

$$\frac{dF_{incident}}{dt} \rightarrow \frac{\Delta Q}{t_{max}}$$

$$T(t) \rightarrow \frac{T_2 + T_1}{2}$$

where the time steps are numbered as $j = 1,2,3,\ldots,N$ and:

T_2 = temperature at the end of the jth time step (°K)

T_1 = temperature at the beginning of the jth time step (°K)

ΔQ = total thermal fluence that is incident on the missile during the jth time step (J/m^2)

Specifically:

$$\Delta Q_j = \Delta CT_j \cdot F_{incident} \tag{2.18}$$

where $F_{incident}$ is defined in equation (2.13) and ΔCT_j is the difference between the fraction of thermal energy emitted up to the beginning and up to the end of the jth time step. The fraction of thermal energy emitted at time t, CT_j, is plotted versus normalized time (t/t_{max}) as the right hand curve in figure 4.

After making the above replacements and solving for T_2, equation (2.17) becomes:

$$T_2 = \frac{[T_1(a - \frac{h \cdot t_{max}}{2}) + h \cdot t_{max} T_{air} + \alpha \Delta Q]}{(a + \frac{h \cdot t_{max}}{2})} \quad (2.19)$$

Thus, the missile skin temperature at the end of time step j can be found using equation (2.19). This equation is next used to calculate the maximum skin temperature reached for a single burst.

Calculating the Maximum Skin Temperature for a Single Burst

The following definitions are needed to describe how the maximum skin temperature is calculated for a single burst:

t = time of missile skin exposure to thermal energy. t is also equal to the time of thermal energy emission by a burst because radiation travels at the speed of light.

t_m = time into missile flight

lt = launch time of the missile. At lt, $t_m = 0$

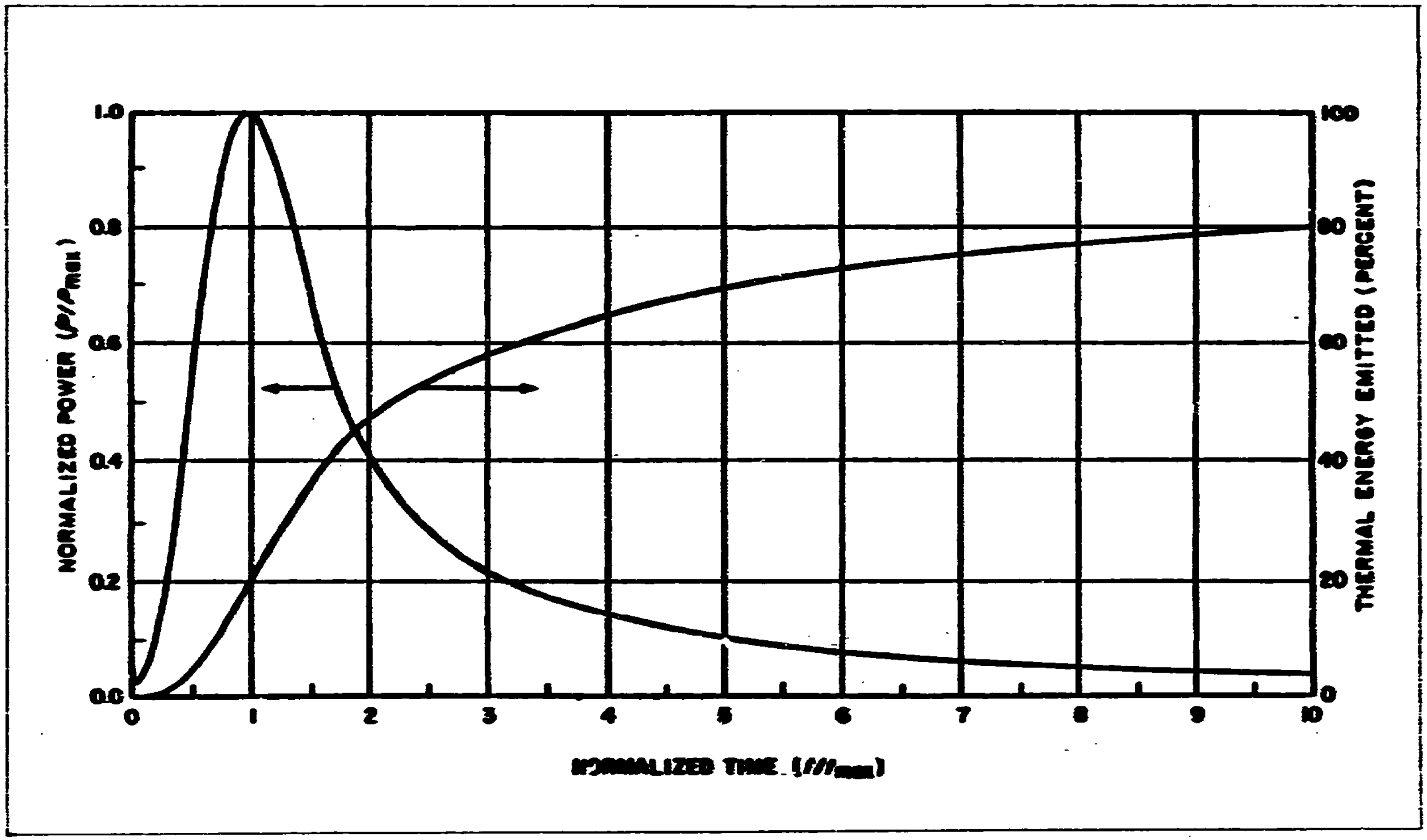


Fig. 4. Fraction of Thermal Energy vs. Normalized Time (Glasstone and Dolan, 1977:311)

tb = time of burst. At tb, $t = 0$

t_0 = time into missile flight when burst occurs
$t_0 = tb - lt$

Thus, two independent time lines are occurring: one defining the missile flight and one defining the thermal radiation emitted by a burst. Figure 5 shows the relation between these time lines and how j is defined for each time step after a burst.

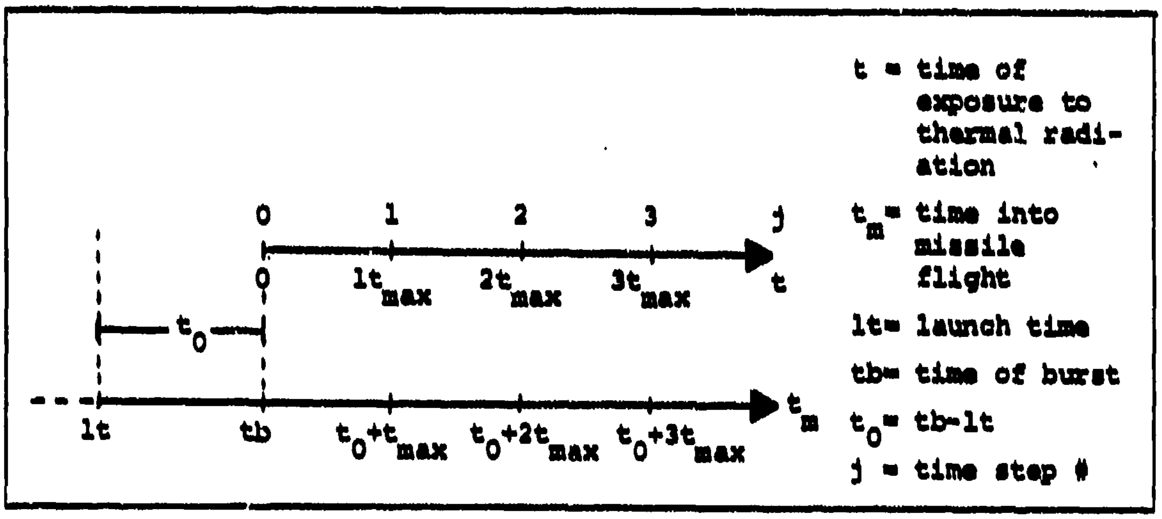


Fig. 5. Single Burst Time Lines

The procedure for determining the maximum skin temperature is an iterative process that begins by calculating T_2 at the end of the first time step ($j = 1$). To do this calculation, the variables T_1, ΔQ, T_{air}, and h must be known. For the first time step, T_1 is assumed to be equal to T_{air} at the missile altitude at the time of burst. The actual value of T_1 would be higher than T_{air}

due to aerodynamic heating, but the assumption is good for a slowly moving missile. ΔQ, T_{air}, and h are dependent on missile and/or burst characteristics that change with time. The missile characteristics are velocity, altitude, down-range-distance, and the flight path angle θ, the angle between the velocity vector and the horizontal plane. The burst characteristics are CT_j and the height of the fireball Hfb_j. If the fireball is assumed to be at the top of the dust cloud, the following equation can be used to calculate Hfb_j (McGahan et al., 1971:40):

$$Hfb_j = 21{,}640.8(W^{.177})[1-(1-t/t_s)^2] \qquad (2.20)$$

where W = yield in megatons and t_s = cloud stabilization time = 240 sec (Bridgman, 1984).

Missile characteristics are calculated at an average time $t_m = t_0 + (j-.5)*t_{max}$, the midpoint of each time step. The altitude and velocity are then used to calculate T_{air} and h. This value for h represents the average amount of convective cooling that occurred during the jth time step. To calculate the slant range (SR), all missile characteristics are needed and the height of the fireball must be known at time $t = (j-.5)*t_{max}$. Once SR is known, the variables CF and τ can be determined. The quantity ΔCT_j is calculated for the time between j and j-1, and is used with SR, CF, and τ to determine ΔQ. Details on how to calculate missile characteristics, burst characteristics, h, t_{air}, SR, CF, and τ are found in Appendices B, C, D, E, and F.

Once the necessary variables are known, T_2 can be calculated for time step j. For the next time step, T_1 is set equal to T_2 and new variables are determined to calculate a new T_2 for that time step. This iteration process is continued until:

1. T_2 is found to be less than T_1. This indicates that the amount of heat removed by convective cooling is now greater than the amount absorbed and therefore the skin temperature will continue to decrease. The maximum skin temperature reached is the current value of T_1; or

2. The number of time steps j equals 10. The process is stopped here because 80% of the thermal radiation from the burst has been emitted, and the value of ΔCT_j would be negligible for the remaining time steps (see figure 4). The maximum skin temperature reached is the value of T_2 at j = 10.

The iteration process to calculate the maximum skin temperature for a single burst is summarized in the algorithm presented in table I. The maximum skin temperature is then used to determine the probability of damage.

Survivability and Aiming Error

For this analysis, the probability of damage is based on the sure safe and sure kill intensities I_{ss} and I_{sk},

TABLE I

ALGORITHM FOR FINDING MaxT FOR A SINGLE BURST

---

1. Set $j = 1$, $T_1 = T_{air}$ at $t_m = t_0$

2. Find missile velocity, altitude, down-range-distance, and θ at $t_m = t_0 + (j-.5)*t_{max}$ (See Appendix B)

   Find fireball height at $t = (j-.5)*t_{max}$ (See equation (2.20))

   Find ΔCT_j for the time j-1 to j (See Appendix F)

3. Calculate h, T_{air}, SR, CF, and τ (See Appendices C, D, and E)

4. Calculate ΔQ using equation (2.18)

5. Calculate T_2 using equation (2.19)

6. If $T_2 < T_1$, then go to step 8
   Otherwise, set $T_1 = T_2$

7. If $j < 10$, then $j = j + 1$ and go to step 2
   Otherwise, go to step 8

8. Maximum skin temperature $MaxT = T_1$

---

where intensity refers to temperature. For aluminum, I_{ss} and I_{sk} are chosen to be 619 °K and 809 °K respectively (Bridgman, 1984). The probability of damage is calculated using the cumulative log-normal distribution function:

$$P_d(I) = \int_0^I \frac{1}{\sqrt{2\pi}\beta' I} \exp[-\frac{1}{2}(\frac{\ln I - \alpha'}{\beta'})^2]dI \qquad (2.21)$$

where the intensity I refers to the maximum skin temperature, MaxT. The parameters α' and β' are calculated using I_{ss} and I_{sk}. By definition, if $P_d(I_{ss}) = 0.98$ and $P_d(I_{sk}) = 0.02$, then from equation (2.21):

$$\alpha' = \frac{1}{2} \ln (I_{ss} I_{sk}) \qquad (2.22)$$

$$\beta' = \frac{1}{4.108} \ln (\frac{I_{sk}}{I_{ss}}) \qquad (2.23)$$

A more detailed discussion of $P_d(I)$, α', and β' is presented in Appendix G.

The probability of damage calculated using the temperature determined from the procedure in table I is the probability of damage from an RV that was assumed to hit at its designated ground zero, or at the center of the targeted silo. However, in reality each RV will have an aiming error that could cause it to land at a point other than its designated ground zero. Since the position of the burst must be known to calculate MaxT, the aiming error needs to be quantified. This is done by dividing the area around a silo into discrete cells whose dimensions are such that each cell has an equal probability of being hit. Although the dimensions of these cells are fixed, the position of the cells around the designated ground zero is not unique. Figure 6 shows the ten cell configuration used in this analysis. It also shows $<\rho_i>$ and $<\theta_i>$, the two parameters that locate the cell centroids such that each cell i is of equal probability. The values of $<\rho_i>$ depend on the circular error probable, or CEP, which is defined as the radius inside of which 50% of the incoming RVs will hit. The method of calculating $<\rho_i>$ and $<\theta_i>$ is described in Appendix G.

The point defined by $<\rho_i>$ and $<\theta_i>$ for cell i is used as ground zero for calculating MaxT. Thus, for a single burst, ten different burst-missile configurations are possible and each configuration will have an associated probability of damage. The total probability of damage for N cells is:

$$P_d = \frac{1}{N} \sum_{i=1}^{N} P_{d_i}(\mathrm{MaxT}) \qquad (2.24)$$

and the probability of survival is then:

$$P_s = 1 - P_d \qquad (2.25)$$

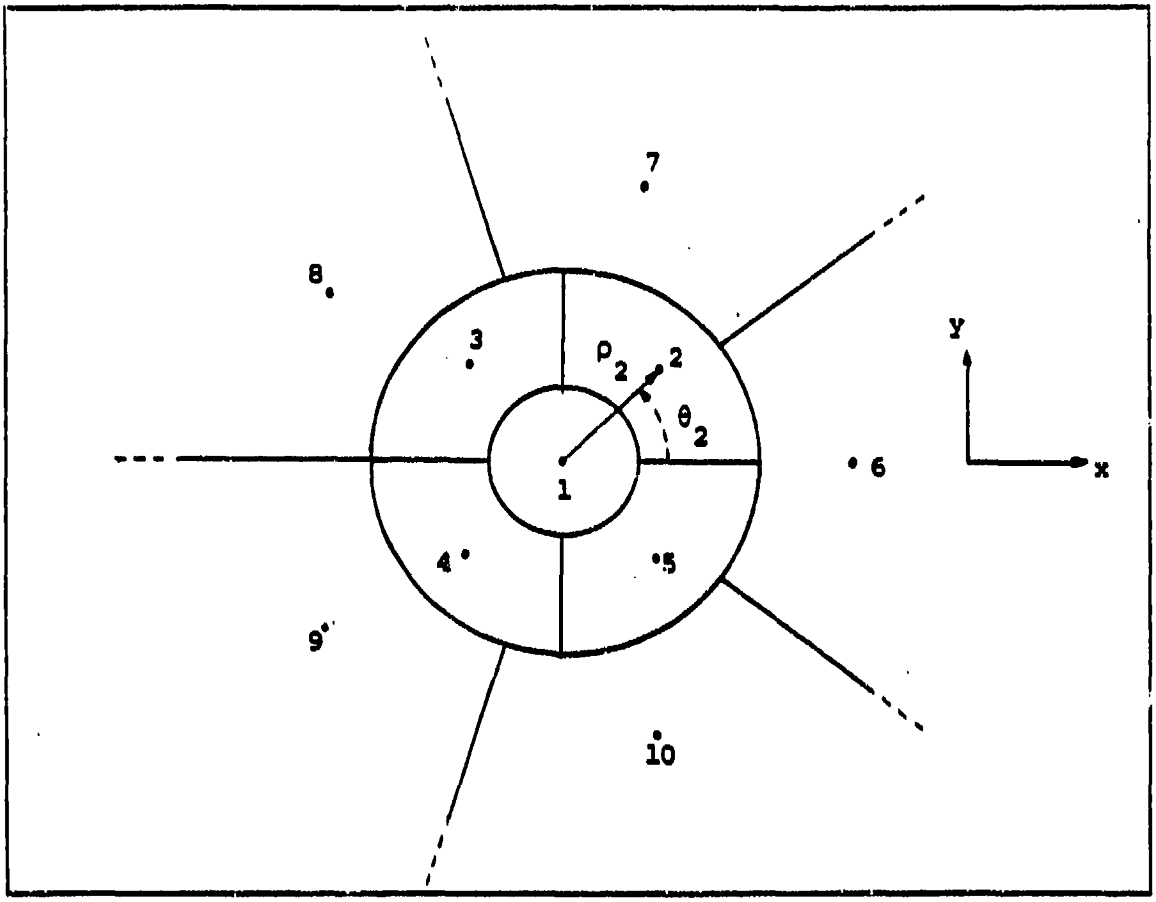


Fig. 6. Ten-cell CEP

The calculation of P_s for a series of bursts depends on whether the noncumulative or cumulative case is being considered.

Calculating the Probability of Survival for a Noncumulative Burst Scenario

For the noncumulative case, each burst is treated as an independent event. Thus, the temperature rise caused by one burst is not affected by the additional thermal radiation from a subsequent burst. Each independent event k has an associated probability of survival $P_s^{\,k}$ calculated from equation (2.25). The total probability of survival for all bursts considered is then:

$$P_s^{\text{Total}} = \prod_{k=1}^{\text{total \# of bursts}} P_s^{\,k} \qquad (2.26)$$

<u>Calculating the Probability of Survival for a Cumulative Burst Scenario</u>

For the cumulative case, each burst is not treated as an independent event. Rather, the amount of thermal energy emitted by each burst will affect T_2 through the term ΔQ in equation (2.19). Equation (2.18) can be rewritten as an expression for ΔQ for a burst k:

$$\Delta Q_j^{\,k} = \frac{tf \cdot y \cdot 4.186 \times 10^{12}}{4\pi} \left(\frac{CF^k \tau^k \; \Delta CT_j^{\,k}}{(SR^k)^2} \right) \qquad (2.27)$$

where y is the same for each burst and the terms in parentheses are burst dependent. SR^k, CF^k, and τ^k are calculated using the position of burst k, and $\Delta CT_j^{\,k}$ is calculated based on the time of burst k.

Figure 7 shows the timing for a series of four 2 megaton bursts. The missile is launched at lt, the first burst occurs at tb_1, and all other bursts occur at two second intervals after tb_1. All calculations for bursts 2, 3, and 4 are referenced to the time steps of burst 1.

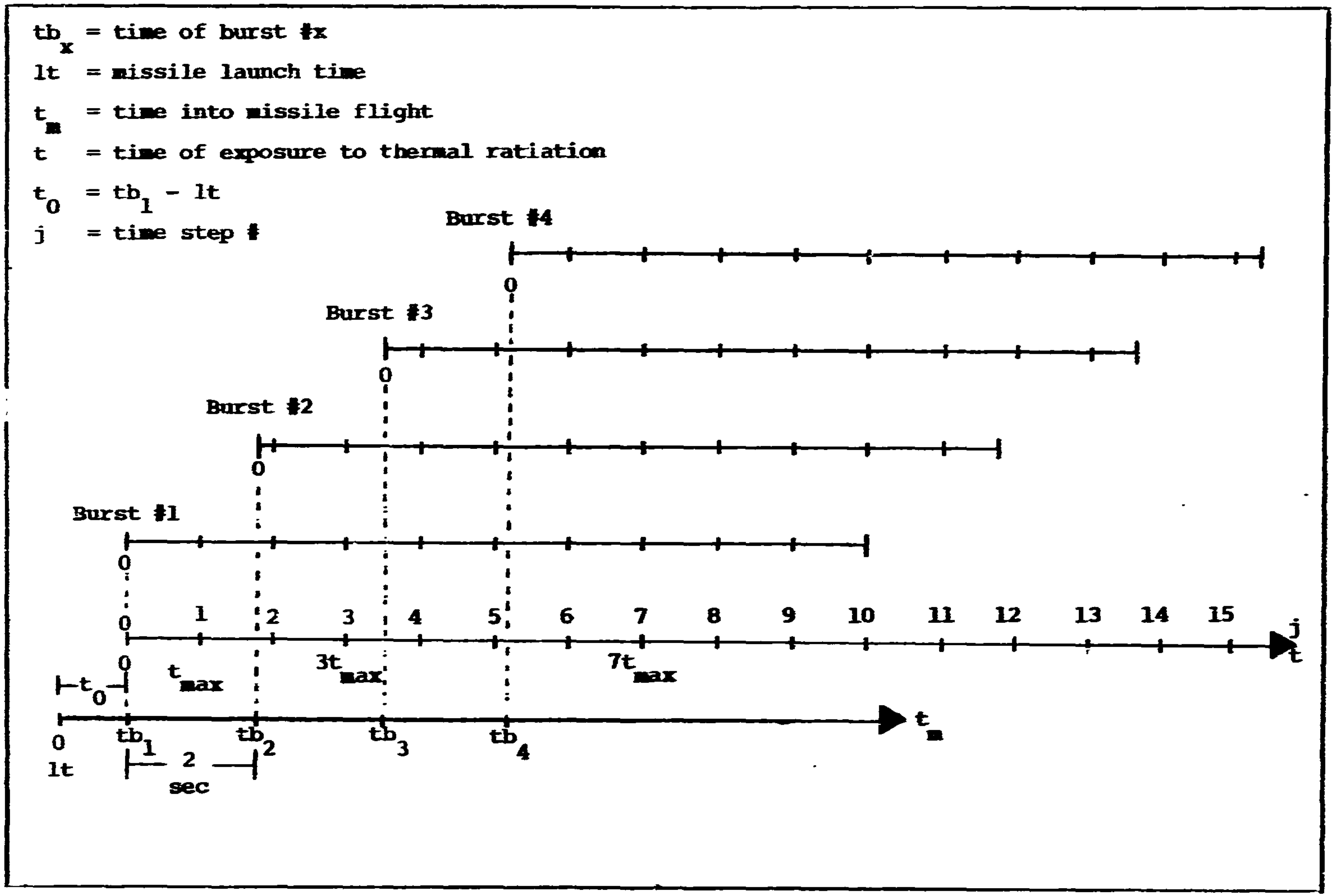


Fig. 7. Multiburst Time Lines

For 2 MT weapons, $t_{max} \cong 1.18$ seconds, so the time steps are shown to be shorter than the interval between bursts.

Because thermal energy is additive, the term ΔQ in equation (2.19) will be a sum of the thermal energy emitted by each burst that has been detonated by time t. For example, at $j = 3$, the missile will be exposed to the thermal energy from two bursts. ΔQ^1 will be calculated as described in table I, using missile characteristics and $Hfb_j^{\,1}$ calculated at the midpoint of the time step, and $\Delta CT_j^{\,1}$ calculated for the time between $j = 2$ and $j = 3$. For the calculation of ΔQ^2 at $j = 3$, the missile characteristics will be the same as those for ΔQ^1 since they are based on missile launch time and therefore do not depend on burst time. However, the slant range will be different because of the different position and fireball height of burst 2. The quantity $\Delta CT_j^{\,2}$ will also be different because burst 2 has not emitted as much radiation as burst 1 for the time between $t/t_{max} = 2$ and $t/t_{max} = 3$.

For a burst k occurring at time tb_k (where $k > 1$), $\Delta CT_j^{\,k}$ will be calculated at time $t' = (tb_1 + j^*t_{max} - tb_k)/t_{max}$. For the time steps where the emission of a burst k ($k > 1$) does not last the full time step, as for $j = 2$, 4, and 6, the calculation of ΔQ^k is slightly altered by finding the missile characteristics at a time that is the midpoint of the first "shortened" time step of burst k. Once all the ΔQ^k's are calculated, they are added to get the total ΔQ, and this term is then used in equation (2.19).

The calculation of P_s^{Total} also changes for the cumulative case. Instead of a series of four 10-cell configurations, now each combination of configurations must be considered. For k bursts, each with a 10-cell CEP, a total of 10^k configurations are possible and therefore 10^k $P_d(MaxT)$'s must be calculated. The total probability of survival for k bursts is then:

$$P_s^{Total} = 1 - \frac{1}{10^k} \sum_{i=1}^{10^k} P_{d_i}(MaxT) \qquad (2.28)$$

The equation for the maximum temperature and the information on how to calculate the probability of survival can now be used to develop a computer program that determines P_s for a series of bursts. However, the final version of this program depends on parameters that are discussed in the next chapter.

III. Problem Parameters

The solution to the problem of determining the maximum skin temperature and the probability of survival for a missile required a number of specific burst and missile conditions and also the choice of certain limiting parameters. These conditions and parameters are summarized in table II. The remainder of this chapter explains the reasons for choosing the parameters listed in table II and also describes how the calculation of the maximum skin temperature was adapted to solution on a computer.

Choosing the Maximum Number of Bursts

The maximum number of bursts affecting the missile, maxb, is an important quantity because it determines the number of burst-missile configurations that need to be considered in order to calculate the probability of survival for the missile. For each burst with a 10-cell CEP, a total of 10^{maxb} configurations are possible and therefore 10^{maxb} maximum skin temperatures must be calculated.

Because the missile is accelerating and rising away from the fireballs, it is possible that an upper limit to the number of bursts exists. Any bursts occurring after this limit would not raise the skin temperature further.

TABLE II

CONDITIONS AND PARAMETERS CHOSEN FOR STUDY

<u>System</u>:
Close-Spaced Basing, or Dense Pack (see figure 1)

<u>Threat Conditions</u>:
Walk attack starting at silo #1 and continuing every 2 seconds on successive silos
Weapon Yield: 2 MT
Height of Burst: 0 m
For surface bursts, t_f = .18
(Glasstone and Dolan, 1977:319)

<u>Missile Conditions</u>:
Missile velocity, altitude, down-range-distance, and flight path angle shown in figure 3 as a function of time
Skin material: Aluminum
$K = .0001\ m^2/s$
$\rho = 2700\ kg/m^3$
$c_p = 900\ J/kg\text{-}^\circ K$
$\alpha = .50$
$I_{ss} = 619\ ^\circ K$
$I_{sk} = 809\ ^\circ K$
Skin thickness: d = .001 m

<u>Probability Conditions</u>:
RV aiming error only for ground zero 10-cell CEP
Probability of damage based on intensity and calculated using the cumulative log-normal distribution function
$P_d(I_{ss}) = .98$
$P_d(I_{sk}) = .02$

<u>Parameters</u>:
Maximum number of bursts considered: maxb = 4
Maximum number of time steps needed: 11
Heat transfer coefficient calculated at $x_m = 5.5$ m

This upper limit can be estimated quickly for any missile by calculating T_2 for a series of bursts and assuming that each RV lands at its designated ground zero (i.e., at the center of cell #1 or a 1-cell CEP). Appendix H shows that the estimated upper limit to the number of bursts was found to be eight, requiring the maximum skin temperature to be calculated 10^8 times. However, in the interest of reducing the amount of computer time, the maximum number of allowed bursts was chosen to be four. Although the probability of survival for a four-burst scenario is an optimistic result, the scenario adequately demonstrates the effects of considering the cumulative versus the noncumulative case.

Choosing the Maximum Number of Time Steps Needed

For the cumulative case, computer time can be saved if the number of time steps necessary to determine the maximum skin temperature is known. The reason for this will be explained later in this chapter. For a single burst, the time considered was t = 10*tmax (see Chapter II). However, for multiple bursts it is possible that the skin temperature could continue to rise after t = 10*tmax, even though the contribution of thermal radiation from the first burst is negligible after this time.

Figure 7 showed the duration of significant thermal radiation four 2 megaton bursts to be approximately 15*tmax or ~17.8 seconds long. However, Appendix I shows that for a 1-cell CEP situation, the maximum skin temperature is reached before $j = 10$. Therefore, to be safe, the maximum number of time steps needed was chosen to be eleven.

Choosing the Position for the Local Heat Transfer Coefficient

The last decision to be made was the choice of x_m, the point along the flat plate representing the missile skin where the local heat transfer coefficient is calculated. McKee chose $x_m = 5.5$ m, the location of the third stage joint on a generic missile (McKee, 1984:7). Comparing values of h for various choices of x_m (see figure E-1) shows that the chosen value of x_m gives heat transfer coefficients that are between those for the minimum and maximum amount of convective cooling. Thus, the location of $x_m = 5.5$ m represents a position where an average amount of convective cooling occurs along the missile skin.

Adapting the Problem to a Computer Program

As explained in Chapter II, determining the probability of survival for a missile exposed to k bursts in the cumulative scenario requires calculating 10^k maximum skin temperatures, one calculation for each geometric configuration due to the 10-cell CEP of each burst. Of the variables used to calculate T_2, ΔQ, or h, only the slant range SR, the transmittance τ, and the term CF depend on the burst location. Thus, to minimize computer time, certain variables can be calculated once and stored for future use.

Specifically, once the missile launch time lt, the time of the first burst tb_1, and the total number of necessary time steps are known, the midpoint of each time step j can be calculated as $t = t_0 + (j-.5)*t_{max}$. Missile characteristics, the ambient air temperature, and the heat transfer coefficient can then be calculated at these times and stored in one-dimensional arrays whose indices refer to the particular time step j. Finally, knowing the total number of bursts to be considered and the time of each burst, the terms ΔCT_j and Hfb_j can be calculated for each time step j and each burst k and stored as a k by j array.

The procedure for calculating the maximum skin temperature for the cumulative case can now be presented using information from the above sections and Chapter II. The algorithm is shown in table III. As mentioned before, the procedure has been simplified by precalculating and storing certain variables for use during the iterative process of calculating T_2.

A FORTRAN77 computer program that follows the algorithm in table III is presented in Appendix K. This program was written on a Corona PC (MS-DOS) and was also used on a VAX 11-780 at the Air Force Institute of Technology. The program calculates the maximum skin temperature for the conditions given in table II and allows the

TABLE III

ALGORITHM FOR FINDING THE MAXIMUM SKIN TEMPERATURE FOR THE CUMULATIVE CASE

---

1. Knowing lt and tb_1, calculate and store missile velocity, altitude, down-range-distance, and θ at $t_m = t_0 + (j-.5)*T_{max}$, where $j = 1, 11$.

   Calculate and store T_{air} and h at the same times.

   Knowing maxb and the times of the bursts, calculate and store ΔQ_j and Hfb_j for each burst and time step.

2. Set $j = 1$, $T_1 = T_{air}$ at $t_m = t_0$, T_2 to be any number greater than T_1, and the number of bursts nb = 1.

3. If $T_2 > T_1$ and $j <= 10$ then:

   a. The current time step is $j = j + 1$; $\Delta Q_j = 0$.

   b. At time j*tmax, determine if another burst has occurred. If so, nb = nb + 1 until nb = 4.

   c. For each burst k that has occurred:

      1. Calculate SR, CF, and τ using the ground zero determined by the cell # of burst k, missile characteristics stored at time j, and Hfb_j for burst k. If burst k has just occurred within the time step, missile characteristics must be re-calculated at the mid-point of that burst's first shortened time step.

      2. Calculate ΔQ^k_j using ΔCT_j for burst k, and add it to the current sum of ΔQ_j.

   d. Calculate T_2 knowing h and the total ΔQ_j for time step j.

   e. If $T_2 > T_1$, then $T_2 = T_1$.

   f. Return to the condition in step 3. If either test fails, go to step 4.

4. MaxT = T_1.

---

user to input the missile number, launch time, number of bursts to be considered ($\leq$ 4), time of the first burst, number of cells to be considered ($\leq$ 10), and the option of considering rising or stationary fireballs.

Also included in Appendix K is a BASIC program that calculates the maximum skin temperature for the noncumulative case. This program follows the algorithm given in table I for a single burst and does not need to precalculate or store any values. The program allows the user to input the same quantities as the program written for the cumulative case.

Chapter IV. Results and Discussion

The main objective of this thesis was to develop a method of calculating the maximum skin temperature of a missile subjected to thermal radiation from more than one fireball at a time. The results of this cumulative case were then compared to the results of the noncumulative case, which considered the effect of thermal radiation from only one fireball at a time. The comparison showed that missiles experienced a much greater temperature rise when subjected to radiation from four fireballs at a time rather than when the four bursts were treated as independent events. Thus, the more realistic case of cumulative thermal effects indicates that targets suffer more thermal damage in a multiburst scenario than previously expected.

The results of the cumulative case are first presented for a 1-cell CEP; i.e., for a perfect hit by an RV on its targeted silo. These results show the temperature rise of the missile skin. Also included are a comparison of considering a rising fireball at the top of a dust cloud to a stationary fireball on the ground, and the effect of decreasing the time step of the calculation to $t_{max}/2$. Next, the cumulative case is compared to the noncumulative case of a missile subjected to four bursts with a 1-cell

CEP. This comparison shows the marked difference in the maximum skin temperature reached in both cases. The scenarios are then extended to the full 10-cell CEP for four bursts to determine which silos are in sure-kill or sure-safe territory. The 10-cell CEP cumulative thermal results are also compared to noncumulative blast results to determine which effect is more lethal. Finally, two topics related to the cumulative results are discussed: the effects of dust shielding and a comparison of 1-cell CEP to 10-cell CEP results.

<u>Temperature Rise for the Cumulative Case (1-cell CEP)</u>

Figure 8 shows the temperature rise of a missile subjected to thermal radiation from a series of four bursts. The temperature plotted, T_2, is the temperature at the end of each time step j, as calculated from equation (2.19). Tabulated data of the curves shown in figure 8 are found in Appendix J. Fireball rise was not considered when calculating these temperature curves because the case of stationary fireballs is less complicated. The missile being considered, #41, was launched at the same time that silo #1 was hit, and each RV landed at its designated ground zero (a 1-cell CEP).

The cumulative effect of thermal radiation from more than one fireball is illustrated by plotting the temperature rise for each burst scenario. Thus, the bottom

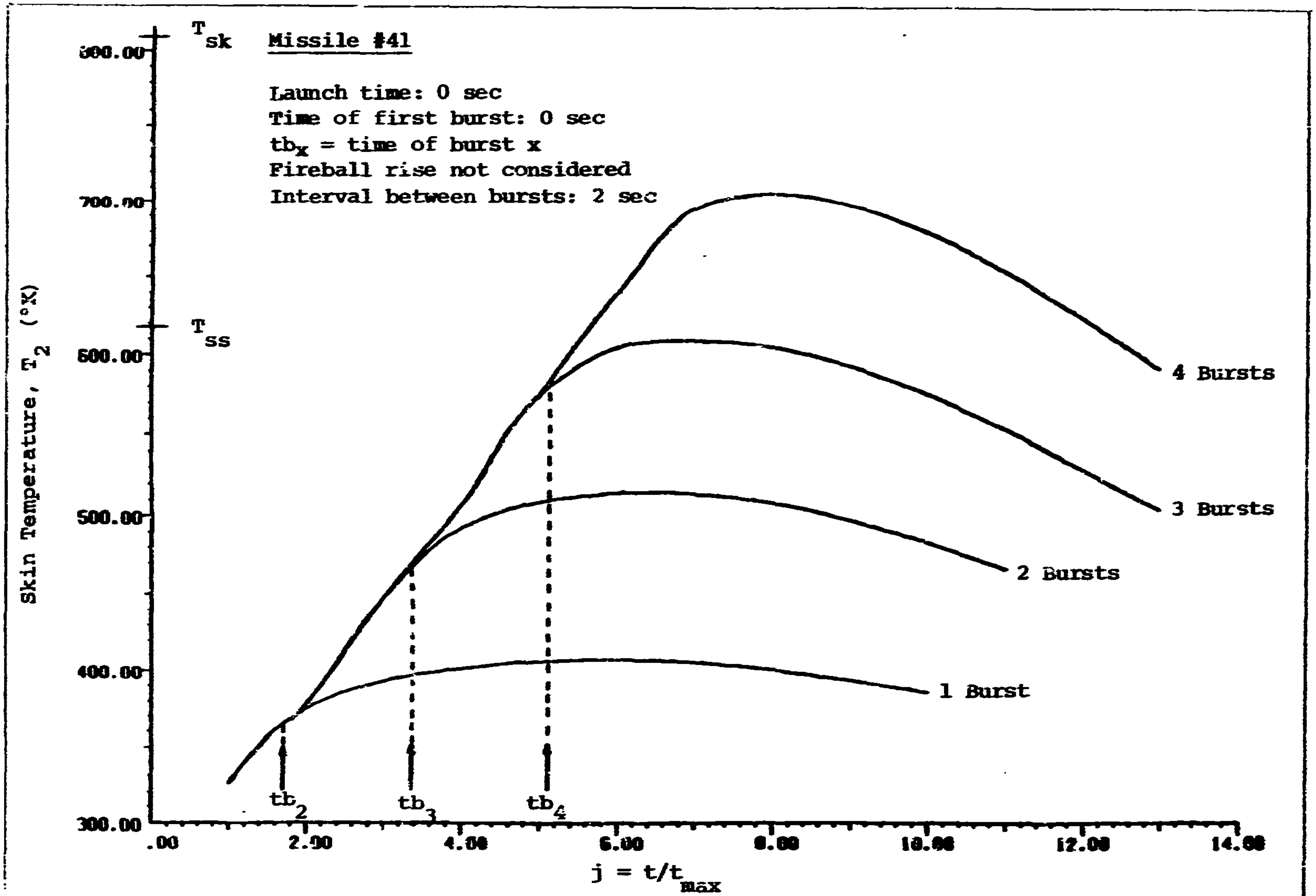


Fig. 8. Temperature Rise for Cumulative Case (1-cell CEP)

curve represents the temperature rise due to one burst, the next curve represents the rise due to two bursts, and so on. The first burst occurred at t = 0, and the times of the three remaining bursts are indicated by arrows. The figure shows that the temperature rise for a series of n bursts ($0 < n \leq 4$) follows the temperature rise for a series of n-1 bursts until the time when the nth burst occurs. At the end of that time step, the temperature is higher than the temperature from n-1 bursts because of exposure to the radiation from the additional burst. Thus, the curve representing the temperature rise due to four bursts is not smooth as it increases but has slight irregularities that indicate the detonation of another burst. Also, the four-burst curve decreases more rapidly from its maximum than the other three curves because the contribution of thermal radiation from bursts 1 and 2, measured by the term ΔQ^k, is small or zero at late times. In addition, convective cooling is more effective because the missile skin is so hot.

The Effect of Fireball Rise. The curves in figure 8 were calculated for the case of fireballs that remained on the ground at ground zero. In reality, a fireball rises along with an expanding dust cloud. For this thesis, the rising fireball was assumed to be located at the top of the dust cloud and centered over ground zero. Therefore, the height of the fireball is equal to the height of the dust cloud as calculated in equation (2.20). When fireball rise is considered, the slant range from burst to missile is calculated from the fireball's height rather than from ground zero.

Table IV compares the temperature rise of missile #41 for both stationary and rising fireballs in a four burst scenario. The data shows that temperatures calculated with fireball rise are generally higher. The difference in the temperature of interest, the maximum temperature (at $j = 8$), is ~5 °K. In terms of probability of damage, P_d(704.5 °K) = .47 and P_d(709.6 °K) = 52 (see Appendix G for an explanation of how to calculate $P_d(I)$). Thus, for missile #41, the case of fireball rise resulted in a slightly higher probability of damage. However, the difference in maximum temperatures would not be readily seen if the values given in Table IV for rising fireballs were plotted in figure 8.

A comparison of maximum temperatures for other missiles is given in table V. The data shows that the difference between maximum temperatures is smaller for missiles launched farther away from the bursts. However, since the case of fireball rise is more realistic, the remaining results presented in this chapter will be those calculated when considering that case.

TABLE IV

TEMPERATURE RISE CONSIDERING STATIONARY AND RISING FIREBALLS

Missile #41
Launch time: 0 sec
Time of first burst: 0 sec
1-cell CEP

Time Step j	Stationary Fireballs T_2 (°K)	Rising Fireballs T_2 (°K)
1	325.4	325.4
2	277.5	377.5
3	447.1	446.8
4	505.7	505.6
5	574.5	575.0
6	638.7	640.4
7	693.6	697.1
8	704.5	709.6
9	695.7	701.9
10	677.8	685.2
11	652.3	660.3
12	622.0	630.2
13	589.7	597.6

TABLE V

MAXIMUM TEMPERATURES CONSIDERING STATIONARY AND RISING FIREBALLS

Missile launch time: 0 sec
Time of first burst: 0 sec
1-cell CEP

Missile #	Stationary Fireballs MaxT (°K)	Rising Fireballs MaxT (°K)
21	2561.6	2615.1
31	1161.3	1175.2
41	704.5	709.6
51	511.5	513.7
61	416.9	418.0
71	366.0	366.6
81	336.9	337.2

The Effect of Halving the Time Step. As mentioned in Chapter II, the finite time step used to solve the differential equation for T_2 was chosen to be t_{max}, the time of the second thermal maximum. However, it is possible that a smaller time step could give more accurate results. To test this possibility, the time step was decreased to $t_{max}/2$. Figure 9 shows that the temperature rise calculated using the reduced time step does not differ much from the temperature rise calculated using t_{max} as the time step (see Appendix J for tabulated data of figure 9). More importantly, the maximum temperature reached in both cases is nearly the same (~709 °K). Therefore, in order to minimize the number of necessary calculations, the finite time step remained as t_{max}.

Comparing the Cumulative to the Noncumulative Case

In the cumulative case, a series of four bursts is treated as one event, and one curve is plotted to illustrate the temperature rise of the missile skin. In the noncumulative case, a series of four bursts is treated as four independent events and therefore four curves are needed to illustrate the temperature rise due to each of the bursts.

The difference between considering four bursts as one event as opposed to four separate events is shown for missile #41 in figure 10 (see Appendix J for tabulated data). The uppermost curve represents the cumulative temperature

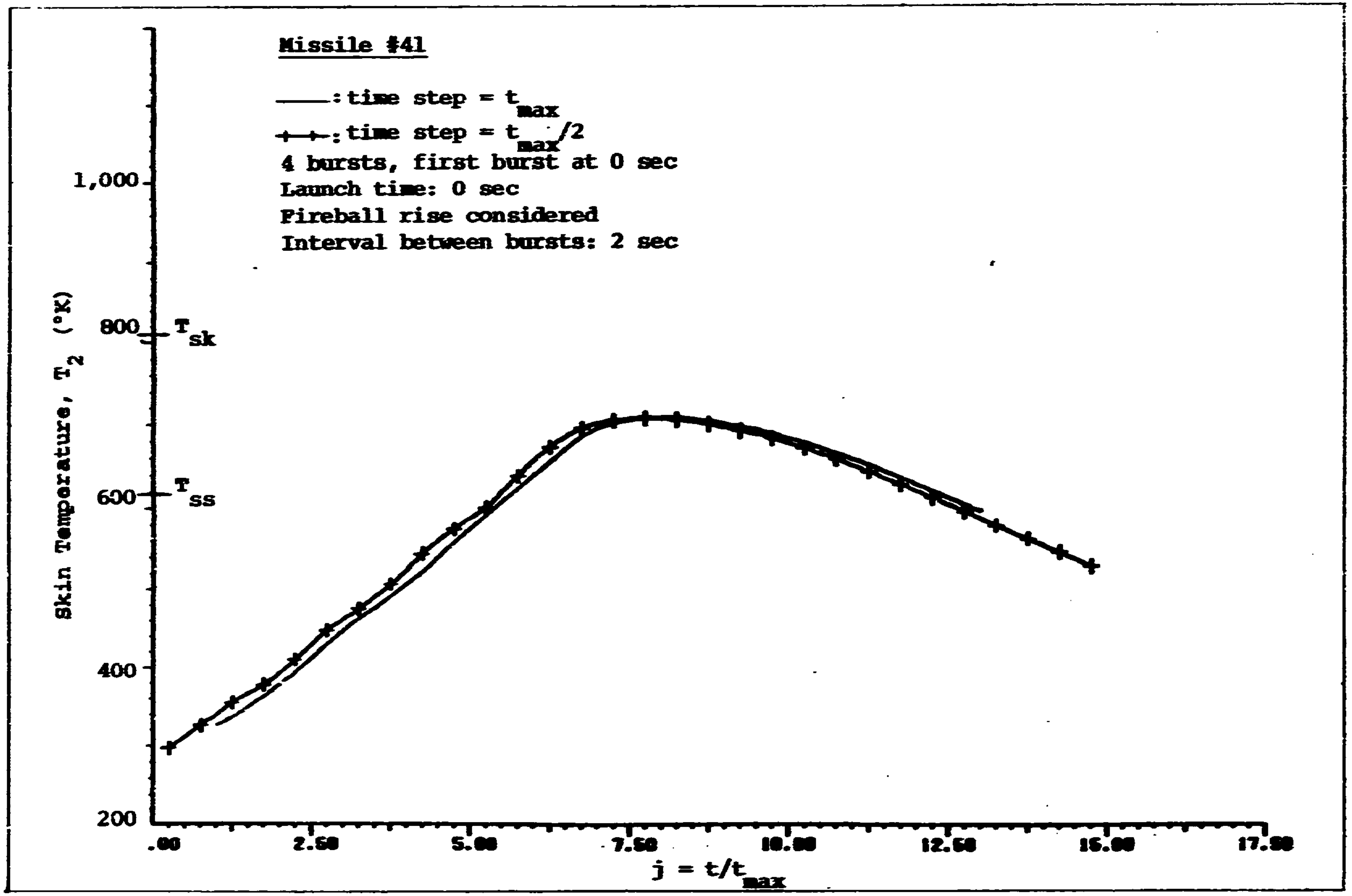


Fig. 9. Comparing Time Steps t_{max} vs. $t_{max}/2$ (1-cell CEP)

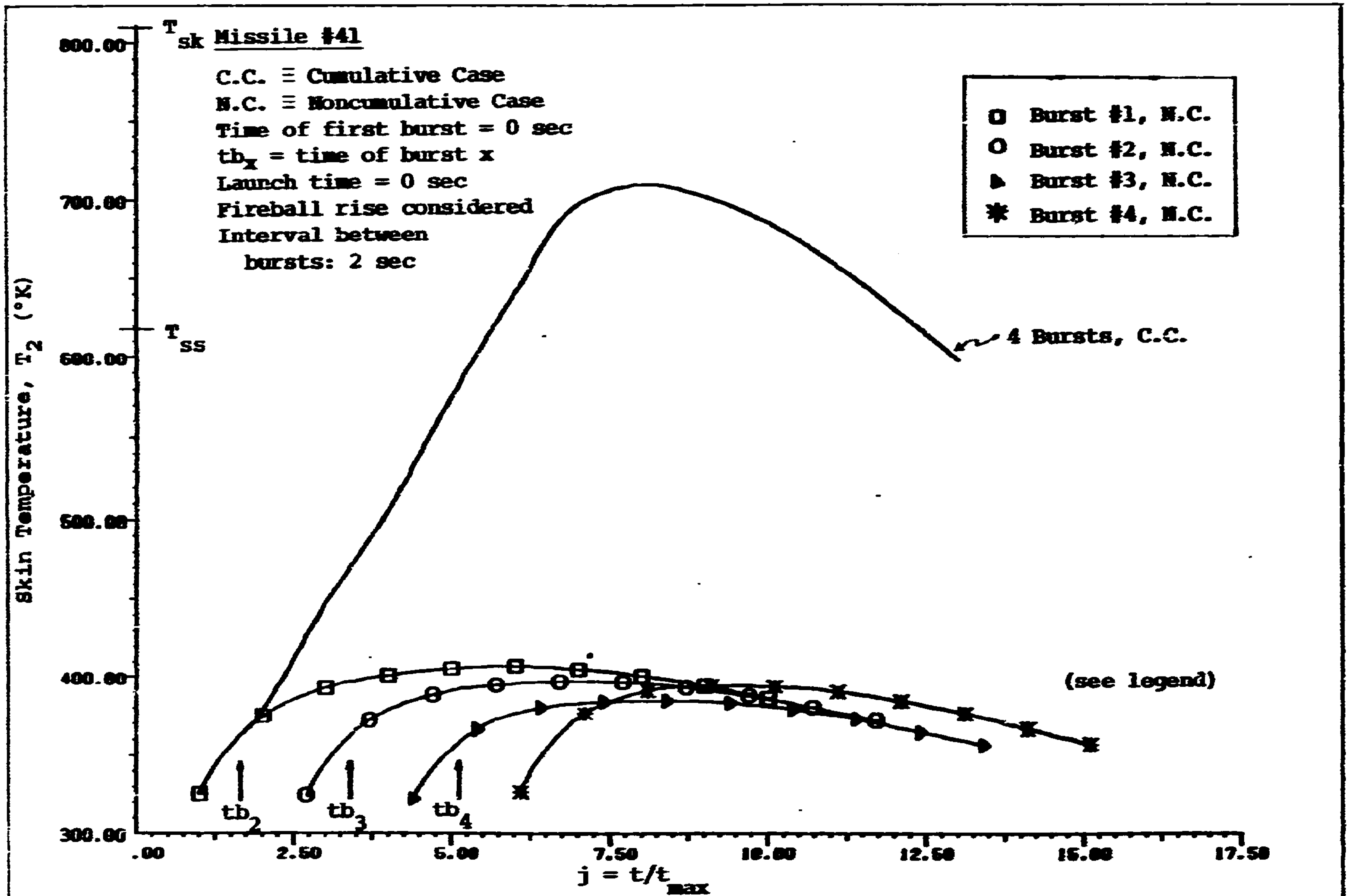


Fig. 10. Temperature Rise Due to Four Bursts: Cumulative vs. Noncumulative Case (1-cell CEP)

rise due to four bursts, while the four bottom curves represent the temperature rise due to each burst separately. In both cases, the bursts are considered to have a 1-cell CEP.

For the noncumulative case, the highest temperature reached is caused by burst #1, since the missile at the beginning of its flight is closest to this burst. However, this maximum temperature of ~407 °K is much lower than the maximum temperature of ~710 °K reached in the cumulative case. Furthermore, the probability of damage for missile #41 in the noncumulative case is 0, while for the cumulative case the probability of damage is ~.52. Assuming that the cumulative case is correct, calculating the temperature rise by considering each burst to be an independent event severely overestimates the missile's chance of survival. The next section further emphasizes this difference by presenting the results using a 10-cell CEP for each burst.

Results Using a 10-cell CEP

As explained in Chapter II, a 10-cell CEP accounts for any possible landing position that an RV can have. Thus, the probability of survival calculated using a 10-cell CEP is a more realistic number than that calculated using a 1-cell CEP unless the value of CEP is very small. Tables VI and VIII present the probabilities of survival for certain missiles in both the cumulative and noncumulative cases. For completeness, values are given for both rising and stationary fireballs.

TABLE VI

PROBABILITY OF SURVIVAL--
NONCUMULATIVE CASE

Missile launch time: 0 sec
Time of first burst: 0 sec
10-cell CEP

Missile #	Stationary Fireballs P_s	Rising Fireballs P_s
25	0	0
26	.575	.545
27	.588	.555
28	.701	.674
29	1	1

TABLE VII

PROBABILITY OF SURVIVAL--
CUMULATIVE CASE

Missile launch time: 0 sec
Time of first burst: 0 sec
10-cell CEP

Missile #	Stationary Fireballs P_s	Rising Fireballs P_s
38	0	0
39	.0616	.0481
40	.0645	.0505
41	.523	.480
42	.507	.464
43	.538	.496
44	.925	.910
45	.927	.913
46	1	1

For this thesis, the sure-safe limit is defined by a probability of damage of .02, and the sure-kill limit by a probability of damage of .98. Any values lower than .02 are rounded to 0, and any values higher than .98 are rounded to 1. Thus, the values of 0 and 1 that appear in tables VI and VII are a result of probabilities of damage that were 1 and 0, respectively. Also, any missile not listed on the two tables has a probability of survival equal to 0 or 1, depending on the missile's silo position.

Another way of presenting the values given in tables VI and VII is by using figure 1 to illustrate the location of the sure-safe and sure-kill regions. This is done in figure 11, with no distinction made for stationary or rising fireballs since the value of P_s will not change the location of the regions. The most obvious characteristic of figure 11 is that the sure-kill region for the cumulative case extends over 2500 m further downfield than the noncumulative case, thus killing thirteen more missiles. Also, the region between the sure-kill and sure-safe limits is larger for the cumulative case, indicating that the probability of survival based on distance is not as steep of a function, as shown in figure 12. The curves drawn in figure 12 indicate the general shape of P_s vs ground range but are not actual functions.

The results of the 10-cell CEP calculations reinforce those of the 1-cell CEP calculations. Both show that

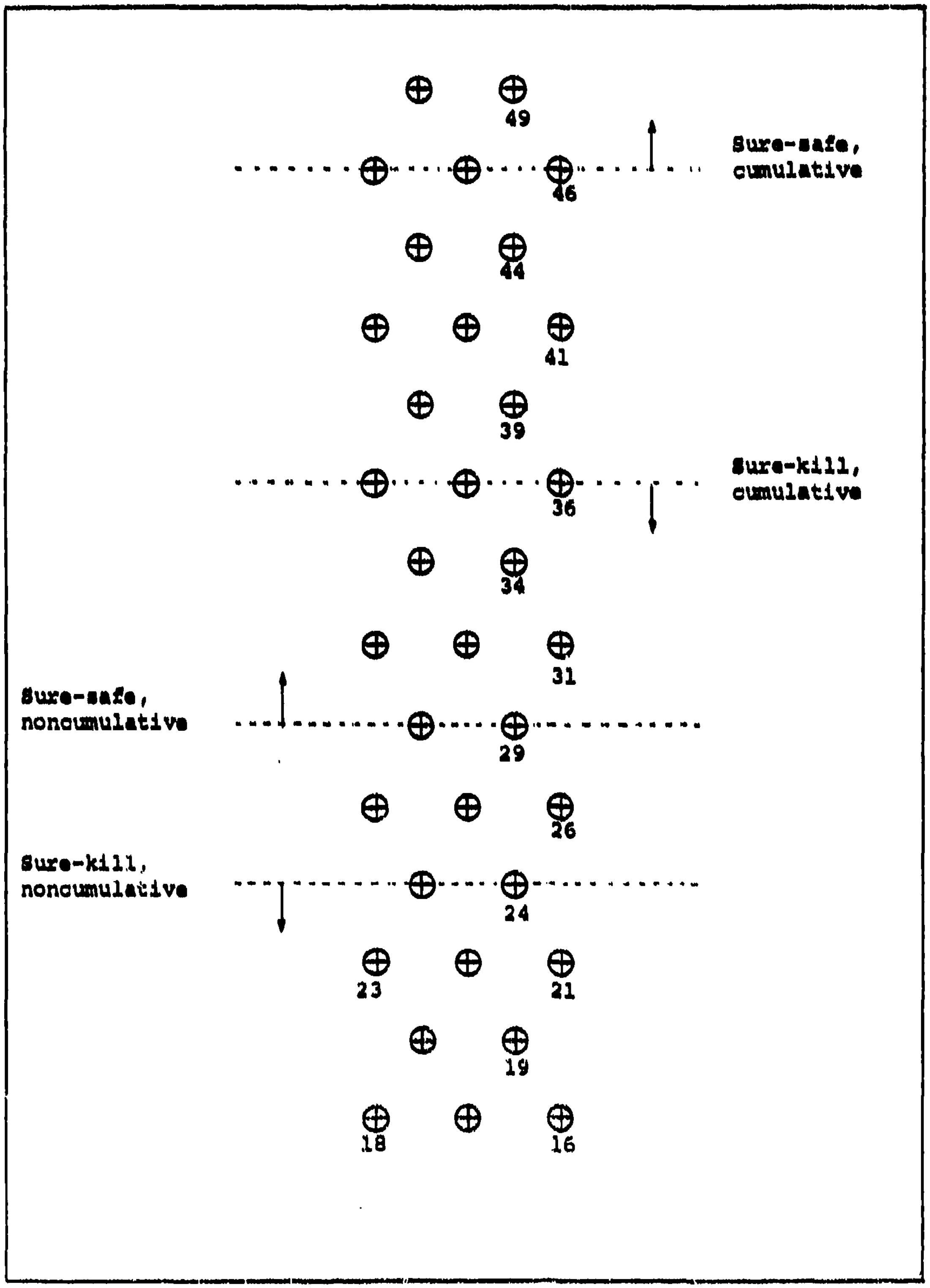


Fig. 11. Location of Sure-safe and Sure-kill Regions

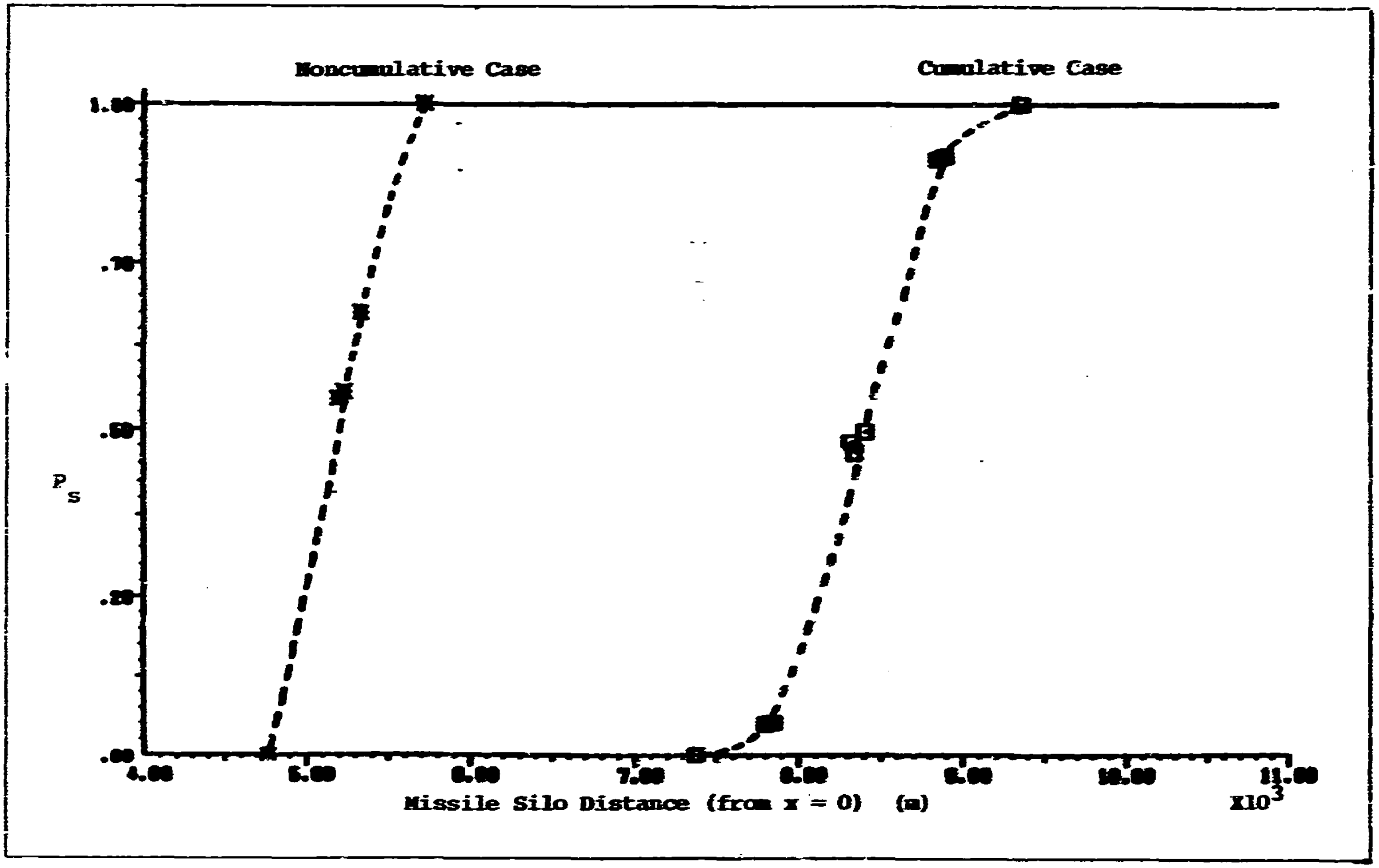


Fig. 12. P_s vs. Distance

treating bursts as independent events does not adequately predict the temperature rise of a missile skin exposed to thermal radiation from a series of bursts. Failing to consider the cumulative effect will underestimate the thermal threat, which could be an expensive oversight in terms of nuclear defense.

Comparing Cumulative Thermal Effects to Blast Effects

In general, blast (overpressure) effects are considered to be more lethal than thermal effects, and in many cases thermal radiation is not viewed as a major threat. Prior to this thesis research, a total survivability analysis of the Dense Pack missile system was performed during NE 6.95, Nuclear Survivability of Systems. The analysis included the same RV walk attack as this thesis, but required a random launch order for the missile field and considered all bursts as independent events. The blast code written for NE 6.95 included a "tail-chase" of the blast wave to the moving target and used sure-safe and sure-kill overpressures of 1.8 and 4.5 psi, respectively. In order to compare cumulative thermal effects with (noncumulative) blast effects, this blast code was modified so that each missile was launched as the first burst occurred. The code was then run to determine the sure-safe and sure-kill regions for blast effects, and the results are given in table VIII.

TABLE VIII

RESULTS OF NONCUMULATIVE BLAST EFFECTS

Missile launch time: 0 sec
Time of first burst: 0 sec
10-cell CEP

Missile #	Probability of Survival
23	0
24	.0717
25	.0956
26	.590
27	.626
28	.750
29	1

Comparing these results to table VI shows that noncumulative thermal and blast effects have nearly the same sure-safe and sure-regions. However, noncumulative blast effects are overwhelmed by the cumulative thermal threat. Although this may not be the case if cumulative blast effects are considered, the results do indicate that in a scenario such as Dense Pack, thermal radiation is more lethal than previously expected if a series of bursts is treated as one event.

Observations on the Cumulative Case

The final section of this chapter presents two topics that were not discussed in the objectives of the thesis but could be of interest. The first topic, dust shielding, is of interest because the dust entrained in the air could greatly reduce the amount of thermal radiation that is transmitted to the missile. The second topic compares the difference between using a 1-cell CEP on a 10-cell CEP.

The Effects of Dust Shielding. A contact surface burst produces a large amount of dust and debris that is carried upward with the rising fireball. This cloud will impede the transmission of thermal radiation to a target by causing additional scattering and absorption of the radiation. The dust cloud created by one fireball may also decrease or even block the thermal radiation emitted by another fireball.

The complex behavior of dust clouds was not included in this study, and therefore the given results would tend to overestimate the temperature rise of the missile skin. However, an attempt was made to determine if dust shielding could be included in the analysis by considering the position of the missile and the fireball. The situation is shown in figure 13.

As figure 13 indicates, the dust cloud is modeled as a cylindrical shape (McGahan, 1971:39-41). Depending on the missile's position, the missile skin will be subjected to either radiation transmitted through the atmosphere or radiation first transmitted through a thickness of dust cloud. For this simplified analysis, the transmission through the dust cloud was considered to depend on the angle ϕ. If the situation was such that the missile was

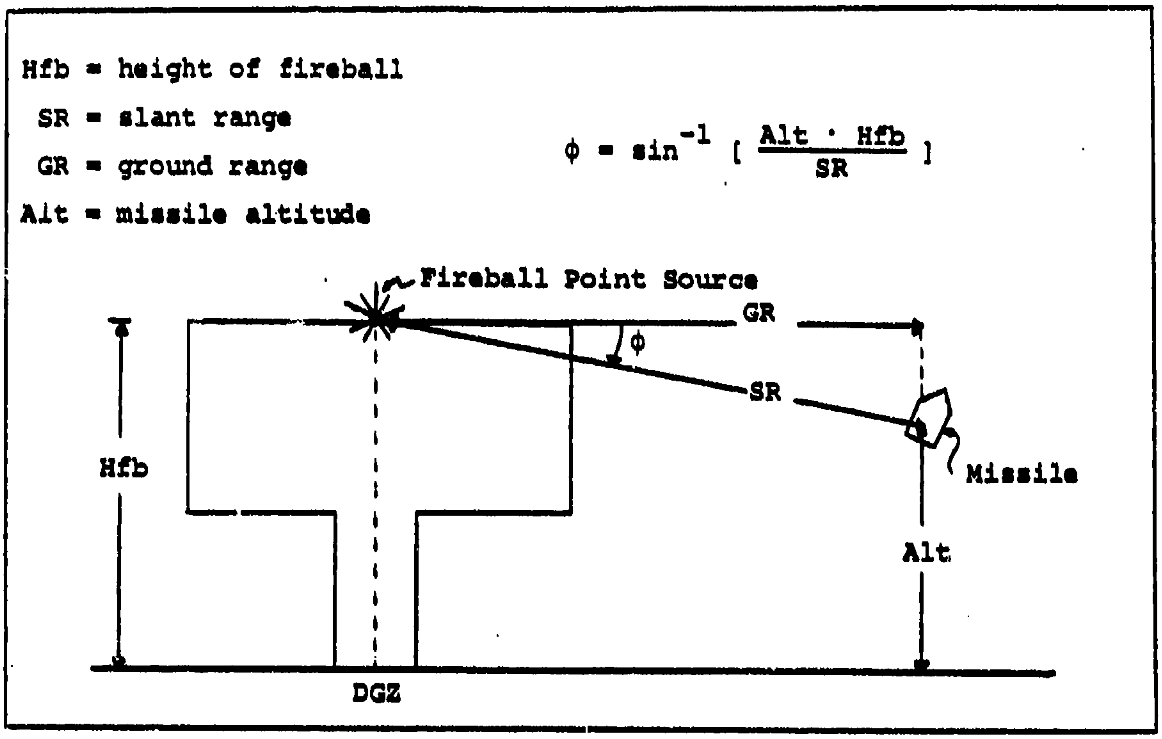


Fig. 13. Dust Shielding

above the top of the dust cloud or ϕ was less than 20°, then the transmittance τ would be the value calculated for the atmosphere (see Appendix C). However, if ϕ were greater than 20°, then τ would be set equal to 0. This limitation only applied to transmission through the cloud from the burst in question, not to transmission through a cloud created by another burst.

Results of the analysis for a 1-cell CEP are given in Appendix L. They show that the dust clouds have no effect on transmittance for any of the missiles listed in table VII. In fact, the transmittance is only affected for missiles whose silos are within ~2750 m of the bursts (i.e., silo #15 and below), but the decrease in temperature rise is not enough to raise the missiles' probability of survival.

Comparing 1-cell CEP and 10-cell CEP Results. As explained in Chapter II, a four burst, 10-cell CEP, cumulative case scenario requires 10^4 calculations of maximum skin temperature to determine the probability of survival of a single missile. Depending on the computer system used, these calculations can take large amounts of computer time and therefore several hours may be needed to obtain results for all of the missiles of interest. In contrast, a four burst 1-cell CEP scenario requires only one maximum temperature to calculate the probability of survival for a single missile, as previously shown in figure 8. Thus, it would be interesting to compare 1-cell CEP results to 10-cell CEP results to determine if a 1-cell CEP can adequately predict a missile's probability of survival. These results are presented in table IX for both stationary and rising fireballs.

The table shows that a 1-cell CEP underestimates the 10-cell CEP P_s for missiles #39 and #40 and overestimates P_s for the remaining missiles. However, the difference between the two values of P_s is less than 10% for missiles #39 and #40 and less than 1% for the others. In addition, the 1-cell CEP results do not change the cumulative sure-kill and sure-safe regions shown in figure 11.

TABLE IX

PROBABILITY OF SURVIVAL, CUMULATIVE CASE--
1-CELL CEP VS 10-CELL CEP

Missile launch time: 0 sec
Time of first burst: 0 sec

Missile #	Stationary Fireballs P_s 1-cell CEP	Stationary Fireballs P_s 10-cell CEP	Rising Fireballs P_s 1-cell CEP	Rising Fireballs P_s 10-cell CEP
38	0	0	0	0
39	.0563	.0616	.0433	.0481
40	.0592	.0645	.0456	.0505
41	.527	.523	.483	.480
42	.510	.507	.466	.464
43	.543	.538	.499	.496
44	.930	.925	.916	.910
45	.933	.927	.919	.913
46	1	1	1	1

Thus, in the interest of reducing computer time, a 1-cell CEP scenario can be used to calculate the probability of survival. The resulting decrease in computer time allows the calculations to be easily performed on a personal computer.

The above conclusion leads to the consideration of calculating the probability of survival for more than four bursts using a 1-cell CEP. According to Chapter III and Appendix H, the maximum number of bursts that will affect the temperature rise of a missile skin was determined to be eight. Table X presents the probability of survival of eight bursts for both stationary and rising fireballs.

TABLE X

PROBABILITY OF SURVIVAL, CUMULATIVE CASE--
EIGHT BURSTS, 1-CELL CEP

Missile launch time: 0 sec Time of first burst: 0 sec		
Missile #	Stationary Fireballs P_s	Rising Fireballs P_s
43	0	0
44	.0503	.0243
45	.0516	.0251
46	.452	.341
47	.437	.327
48	.460	.349
49	.884	.827
50	.886	.830
51	1	1

The results are also shown in figure 14, which compares the sure-kill and sure-safe regions for four and eight bursts. The sure-kill region for eight bursts is extended ~1000 meters down-range, killing an additional five missiles. Thus, the consideration of eight bursts further illustrates the fact that cumulative thermal effects are much more lethal than noncumulative thermal and blast effects.

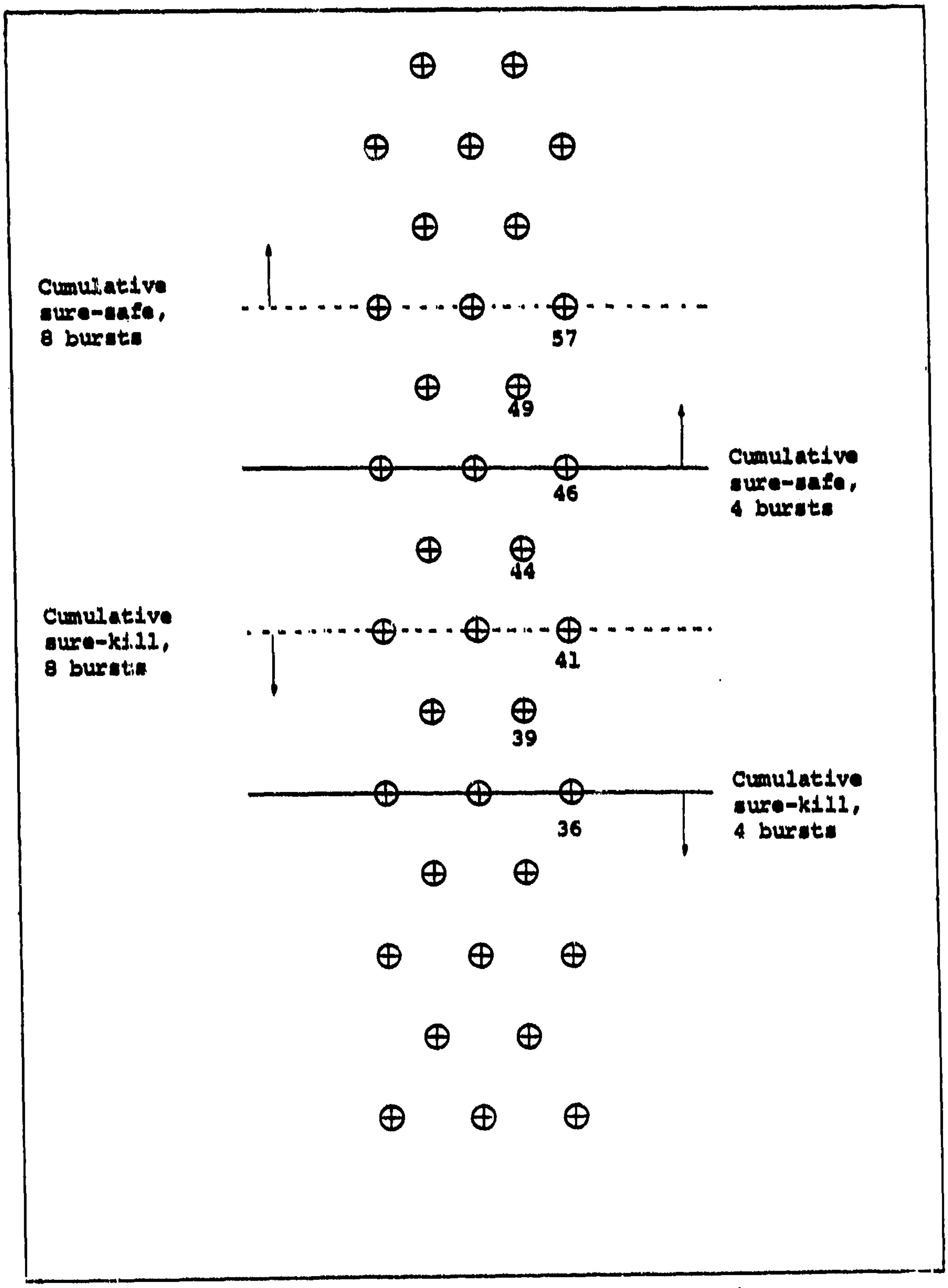


Fig. 14. Sure-safe and Sure-kill Regions,
4 and 8 Bursts (1-cell CEP)

Chapter V. Conclusions and Recommendations

Conclusions

Based on the results given in Chapter IV, the following conclusions are drawn:

1. The calculated value of the maximum skin temperature for a missile exposed to a series of bursts depends on the way in which the bursts are treated. Considering a series of bursts as one event (cumulative) gives a higher temperature than considering each burst as an independent event (noncumulative). A higher temperature yields a lower probability of survival for the missile.

2. Maximum temperatures calculated when considering fireball rise are higher than those calculated for stationary fireballs. However, the difference between the two cases is not large enough to distinguish between sure-safe and sure-kill regions for either case. Therefore, fireball rise as treated in this thesis does not have a significant effect on the results.

3. The difference in maximum temperatures between the cumulative and noncumulative cases extends the sure-kill region of the cumulative case further downfield, making the cumulative case more lethal. Specifically, for a four burst scenario with each missile launched as the first

burst detonates, the cumulative case kills thirteen more missiles than the noncumulative case.

4. Cumulative thermal effects are also shown to be more lethal than blast effects, which contradicts the current opinion that blast is the primary kill mechanism. Based on this conclusion and conclusion #3, the failure to consider the cumulative case will underestimate the thermal threat and could be an expensive oversight in terms of nuclear defense.

5. The results given are considered to be conservative because the analysis did not account for any thermal radiation shielding by dust. However, dust shielding cannot be adequately modeled using simple geometry between the missile and the dust cloud.

6. In order to reduce computer time, a 1-cell CEP rather than a 10-cell CEP can be used to calculate the probability of survival. If this is done, the maximum number of bursts considered can be raised to eight. This results in five more missiles being killed and further emphasizes the importance of considering the cumulative case.

Recommendations

Based on the assumptions presented in Chapter I and the observations made during this study, the following recommendations are made for further study:

1. Several assumptions made during the study ignored the effect of dust shielding on the amount of radiation transmitted to the target. The expression derived for τ was based on data given for a clear atmosphere and therefore overestimated the transmittance. The curve CT (figure 4) used to determine the fraction of thermal radiation emitted at the time t was derived for an air burst, and would be lower for a surface burst. Finally, the assumption that the fireball is located at the top of the dust cloud and the simplified geometric analysis of dust shielding did not consider any of the detailed physics involved in dust cloud formation. Since the amount of dust lifted into the air by a series of 2 MT bursts could be quite large, the effect of the dust on the transmittance of thermal radiation should be investigated.

2. The fireball was assumed to be an isotropic point source. This is a good assumption for distances that are much greater than the fireball diameter. In this study, the length of the missile field was 20,300 m. However, the maximum fireball diameter for a 2 MT contact surface burst is expected to be 3700 m (Glasstone and Dolan, 1977:71). Thus, the distances between burst and target are less than five times the fireball diameter. It should be determined if the distances are great enough to justify the assumption of a point source. If not, the effects of a finite source

on the results presented in Chapter IV should also be investigated.

3. This study did not consider the synergistic effects of heat and blast. It is possible that the increase in temperature would sufficiently weaken the missile's skin so that a lower overpressure would cause the same amount of damage that a higher overpressure caused when considered alone. Synergistic effects should be studied to determine if thermal radiation poses an additional threat by making a system more vulnerable to blast effects.

4. The program written for this study is limited because it was written for a specific missile system and a four burst scenario. A more general program would be easier to adapt to other situations and/or materials.

Appendix A. An Expression for the Thermal Diffusion Time

In Chapter II, the expression for the time required for heat to be conducted through a slab of thickness d with thermal diffusivity K was given as:

$$t_{diff} = d^2/K \tag{A.1}$$

This expression can be derived from the one-dimensional heat transfer equation (Holman, 1972:102)

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{K}\frac{\partial T(x,t)}{\partial d} \tag{A.2}$$

using the method of dimensional analysis.

As the name implies, dimensional analysis involves the algebraic manipulation of the dimensions of physical quantities to provide information about the physical processes involved. The basis of dimensional analysis is the simple principle of dimensional homogeneity, which states that an equation is complete only if the dimensions of each term are the same, or homogeneous (Parkhurst, 1964:16). Equation (A.2) can be shown to be complete by noting that infinitesimals possess the dimensions of the physical elements they represent (Parkhurst, 1964:21). Therefore, each expression on either side of equation (A.2) has the dimensions of θ/L^2, where θ indicates a temperature unit and L indicates a unit length.

An important application of the principle of homogeneity is the derivation of an unknown quantity based on that quantity's functional dependence on the physical parameters involved in the problem (Parkhurst, 1964:16). This derivation is commonly called the indicial method, and is illustrated here by deriving an expression for t_{diff} from the following terms appearing in equation (A.2):

term	description	fundamental units
$\partial T => \Delta T$	temperature differential	θ
$\partial x => \Delta x$	distance differential	L
K	thermal diffusivity	L^2/t
$\partial t => \Delta t => t_{diff}$	diffusion time	t

The first step in the indicial method is to write the unknown quantity as a product of terms to unspecified powers or indices:

$$[t_{diff}] = [\Delta T^a \ \Delta x^b \ K^c] \qquad (A.3)$$

The dimensions of each term are then substituted into the above equation to obtain:

$$[t] = [(\theta)^a \ (L)^b \ (L^2/t)^c]$$

or $$[t] = [(\theta)^a \; (L)^{b+2c} \; (t)^{-c}] \tag{A.4}$$

The indices of each fundamental unit on the left- and right-hand side are then equated to obtain j simultaneous equations, one for each unit:

$$0 = a$$

$$0 = b + 2c$$

$$1 = -c$$

In general, for n indices, if k of the j equations are independent, then any k of the indices can be solved for in terms of n-k other indices. For this case, all of the indices can be solved for to obtain:

$$a = 0$$

$$b = 2$$

$$c = -1$$

Therefore, the expression for t_{diff} can be found by substituting the above values for a, b and c into equation (A.3) to obtain:

$$t_{diff} = \Delta x^2 \; K^{-1}$$

or, since $\Delta x = d$:

$$t_{diff} = d^2/K \tag{A.5}$$

Appendix B. Missile Characteristics

The missile's velocity v, altitude z, down-range-distance (from the first row of missiles) drd, and flight path angle θ must be known at a given time t in order to calculate the slant range SR, correction factor CF, transmittance τ, ambient air properties, and the heat transfer coefficient h. The missile characteristics are shown as a function of time in figure 3. In this figure, the down-range-distance curve refers only to the ground distance from the missile silo, not from the first row of missiles.

In order to use the information on the graph in a computer program, each curve was reduced to a data file of values read from the curve at every second. Values were taken from t = 0 to t = 50 seconds for the velocity, altitude, and down-range-distance data files, and from t = 0 to t = 70 sec for the flight path angle data file. The contents of these files are presented in table B-I. At the beginning of the computer program, the data files are read into the arrays VDATA(51), ZDATA(51), XDATA(51), and ANGDAT(71).

The missile's velocity, altitude, and down-range-distance at any time t can now be determined in one of two ways:

TABLE B-I

DATA FILES USED IN COMPUTER PROGRAM*

File "VELDATA.TXT" Missile Velocity at each Second t = 0 to 50	File "ALTDATA.TXT" Missile Altitude at each Second t = 0 to 50	File "DRDDATA.TXT" Missile Down-Range-Distance at each Second t = 0 to 50	File "DEGDATA.TXT" Missile Angle at each Second t = 0 to 70	
v (ft/sec)	z (ft)	drd (ft)	θ (°)	
0	0	0	90	50
20	50	25	89.9	49
50	100	50	89.5	48.5
70	200	75	88.7	47.8
100	500	100	85	47
140	600	200	80	46.2
170	1000	300	76	45.5
250	1200	400	72	45
300	1600	500	69.5	44
350	2000	700	67.5	43.8
400	2500	1000	66	43.2
470	3000	1100	64	42.8
520	3500	1500	62.5	42
580	4000	1900	61	41.5
650	4500	2100	60	41
700	5000	2500	58.5	40.5
780	5600	2900	57	40
850	6400	3100	56	39.5
950	7000	3500	54.5	39
1000	7800	4000	53.5	38.8
1100	8500	4500	52.5	38
1160	9500	5000	51.5	37.8
1250	10500	5600	50.8	37.2

TABLE B-I--Continued

File "VELDATA.TXT" Missile Velocity at each Second t = 0 to 50	File "ALTDATA.TXT" Missile Altitude at each Second t = 0 to 50	File "DRDDATA.TXT" Missile Down-Range-Distance at each Second t = 0 to 50	File "DEGDATA.TXT" Missile Angle at each Second t = 0 to 70
v (ft/sec)	z (ft)	drd (ft)	θ (°)
1300	11500	6500	37
1400	12500	7200	36.5
1470	13500	8000	36
1550	14500	9000	35.5
1630	16000	10000	35.2
1700	17000	11000	34.8
1800	18500	12000	34.5
1900	20000	13500	34.2
1970	21000	15000	33.8
2060	23000	16500	33.5
2160	24000	18500	33.2
2260	26000	20000	32.8
2360	27800	22000	32.5
2470	29500	23500	32.2
2560	31500	25550	32
2660	33000	28000	31.8
2770	35000	30000	31.6
2870	37000	32000	31.5
2970	39000	34500	31.4
3070	41000	37000	31.2
3200	43000	39000	31
3300	45000	42000	30.9

TABLE B-I--Continued

File "VELDATA.TXT" Missile Velocity at each Second t = 0 to 50	File "ALTDATA.TXT" Missile Altitude at each Second t = 0 to 50	File "DRDDATA.TXT" Missile Down-Range-Distance at each Second t = 0 to 50	File "DEGDATA.TXT" Missile Angle at each Second t = 0 to 70
v (ft/sec)	z (ft)	drd (ft)	θ (°)
3400	47000	44000	30.8
3500	49000	46000	30.7
3600	51500	49000	30.6
3700	54000	52000	
3800	56000	55000	
3900	58000	58000	

*Values taken from figure 3.

1. If t is $\leq$ 50 seconds, then v, z, or drd can be obtained by linear interpolation, where:

$t_1 < t < t_2$; t_1 and t_2 are whole numbers and $t_2 - t_1 = 1$

$y_1 < y < y_2$; $y \equiv v$, z, or drd

$$\text{and} \quad y = (t - t_1)(y_2 - y_1) + y_1 \quad \text{(B.1)}$$

2. If t > 50 seconds, then

$$v = 105t - 1{,}350 \quad \text{(B.2)}$$

$$z = 2{,}460t - 65{,}000 \quad \text{(B.3)}$$

$$drd = 3{,}700t - 127{,}000 \quad \text{(B.4)}$$

Since the curves in the figure are in terms of feet, all parameter values must be converted to meters. Also, the values of drd must be modified to reflect the missile's distance from x = 0, or the first row of missiles. This is accomplished by adding the missile silo's x-coordinate to drd:

$$drd = drd{*}.0348 + S_x(Mn) \quad \text{(B.5)}$$

where $S_x(Mn)$ = Silo x-coordinate of missile Mn.

The missile's flight-path-angle θ is found in a similar manner:

1. If t $\leq$ 70 seconds, then

$$\theta = (t - t_1)(\theta_2 - \theta_1) + \theta_1 \quad \text{(B.6)}$$

2. If $t > 70$ seconds, then:

$$\theta = 30.6 - .1(t-70) \tag{B.7}$$

Equation (B.7) is a linear extension of the curve in figure 3 past $t = 70$ seconds. The value of θ is then converted to radians.

Appendix C. Calculating the Slant Range, Correction Factor, and Transmittance

The calculation of SR, CF, and τ are presented in one appendix because the latter two terms depend on the value obtained for the slant range. However, the procedures used to calculate each term are different and will be presented in separate sections within this appendix.

Calculating the Slant Range

The slant range is generally defined as the distance from the burst to the target. In this report, the slant range is the distance from the fireball point source to the location of the missile as defined in figure 3. If fireball rise is being considered, the expression for SR must take the changing position of the point source into account. The slant range for any burst-missile combination is shown in figure C-1. The missile has no velocity in the y direction and thus travels straight north along the x-axis. According to figure C-1:

$$SR = [GR^2 + (alt - Hfb)^2]^{0.5} \qquad (C.1)$$

Hfb is calculated using the VORDUM equation for the top of the dust cloud (McGahan et al., 1971:40):

$$Hfb = 21,640.8w^{.177}\ [1-(1-t/t_s)^2] \qquad (C.2)$$

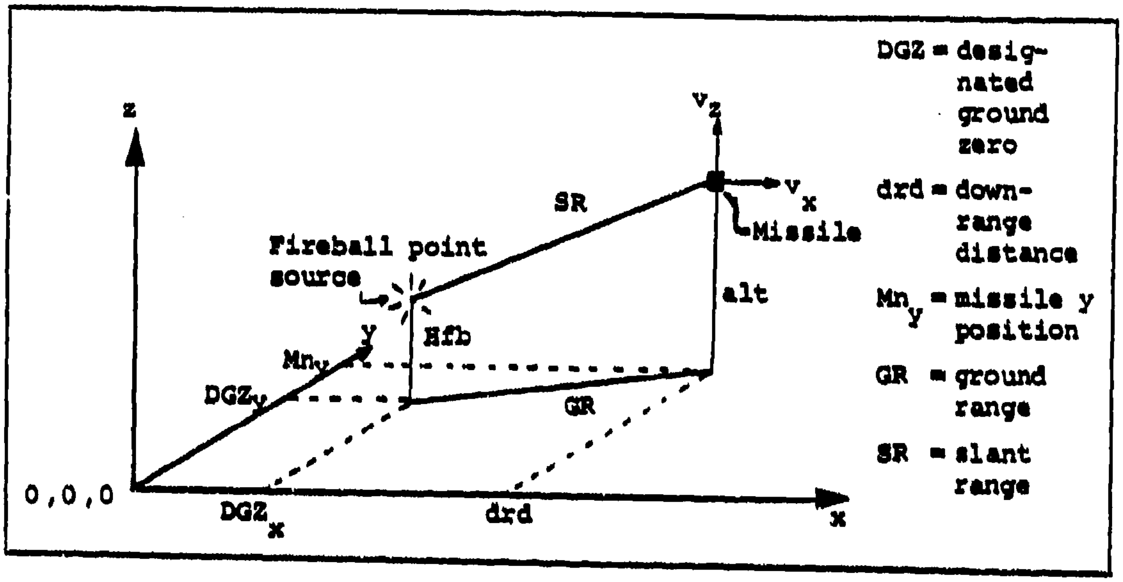


Fig. C-1. Slant Range from Burst to Target

where w = yield in megatons, and t_s = 240s (Bridgman, 1984), the time of cloud stabilization.

For an RV that lands at its designated ground zero (the targeted silo), the expression for the ground zero range is:

$$GR = [(Mn_y - DGZ_y)^2 + (drd - DGZ_x)^2]^{.5} \qquad (C.3)$$

However, if the RV has an aiming error associated with its landing position, then the expression for GR must account for the displacement from designated ground zero. This displacement is quantified using the 10-cell CEP described in Appendix J. The situation for calculating GR for a cell is shown in figure C-2.

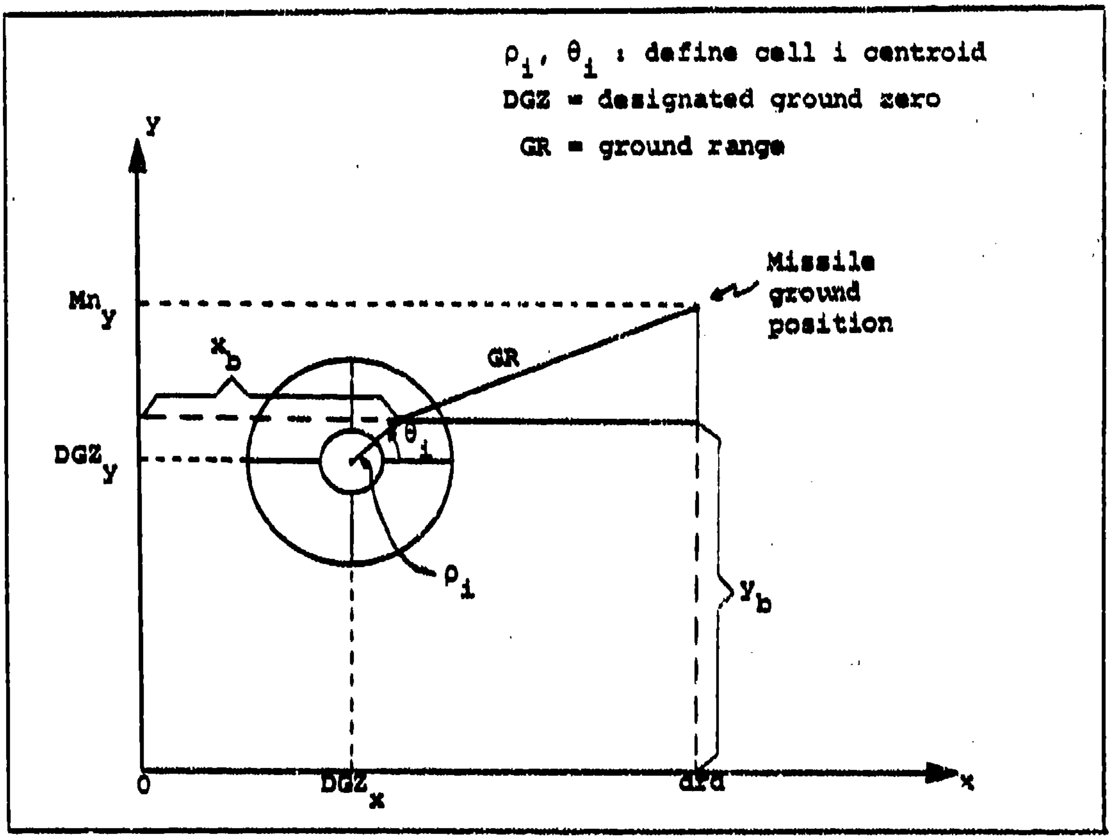


Fig. C-2. Ground Range from Cell i

From figure C-2, a more general expression for ground range is given by:

$$GR = [(Mn_y - y_b)^2 + (drd - x_b)^2]^{.5} \qquad (C.4)$$

where $x_b = DGZ_x + \rho_i \cos \theta_i$ and $y_b = DGZ_y + \rho_i \sin \theta_i$

For the middle cell (a direct hit), equation (C.4) reduces to equation (C.3).

Calculating the Correction Factor

In Chapter II, the correction factor CF was defined as sin ψ where ψ is the angle between the missile skin surface and the slant range vector. The correction factor is needed to calculate the amount of thermal radiation that is incident perpendicular to the missile skin's surface. CF can be calculated by recognizing that cos ψ is defined by the dot product of the slant range and velocity vectors as shown in figure C-3:

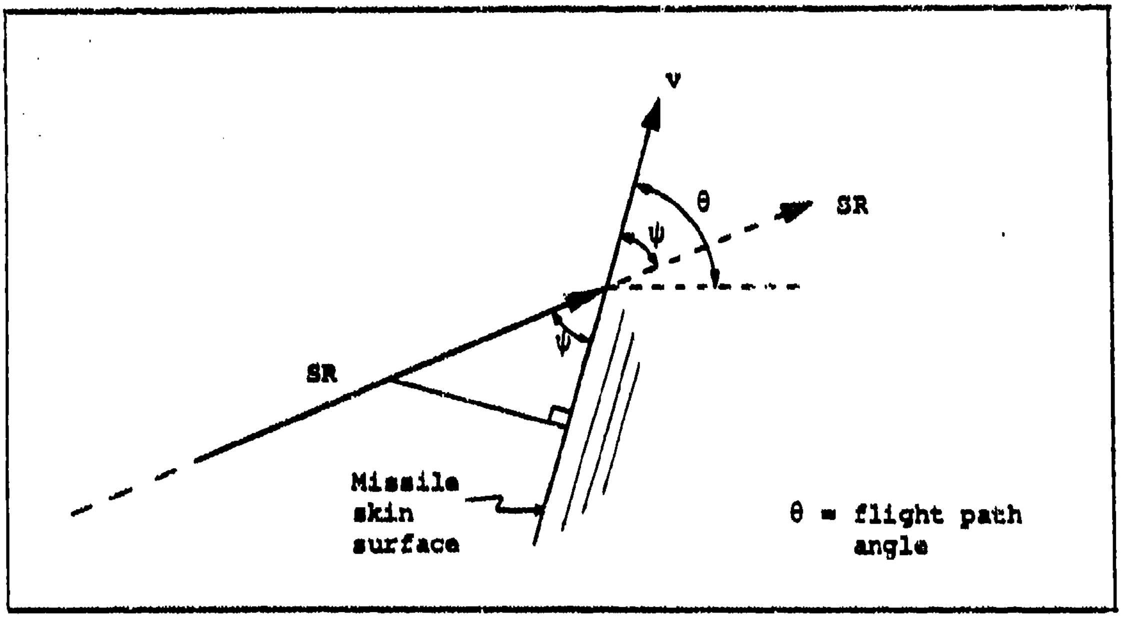


Fig. C-3. How to Calculate CF

By defintion of the dot product:

$$\cos \psi = \frac{SR_x v_x + SR_z v_z}{SR \cdot v} \tag{C.5}$$

where $v_x = v \cos\theta$

$$v_z = v \sin\theta$$

$$SR_x = GR_x = drd - x_b$$

$$SR_z = alt - Hfb$$

The product of y components does not appear in equation (C.5) because $v_y = 0$. It should also be noted that negative values of SR_z are possible if alt < Hfb. Although a negative value does not mean much physically, it is necessary to obtain the correct answer geometrically. Once $\cos\psi$ is known, CF can be determined from:

$$CF = \sin\psi = (1 - \cos^2\psi)^{.5} \qquad (C.6)$$

<u>Calculating the Transmittance</u>

The transmittance τ is defined as the fraction of direct and scattered radiation that is transmitted to the target (Glasstone and Dolan, 1977:316). Because τ is a complex function of atmospheric conditions and distance, a detailed analysis of the correct values of τ for the conditions of the Dense Pack scenario was not conducted. Instead, τ is assumed to behave as shown in figure C-4 (Glasstone and Dolan, 1977:318). The limitations of this figure are as follows:

1. The target is such that scattered radiation is received from all directions.

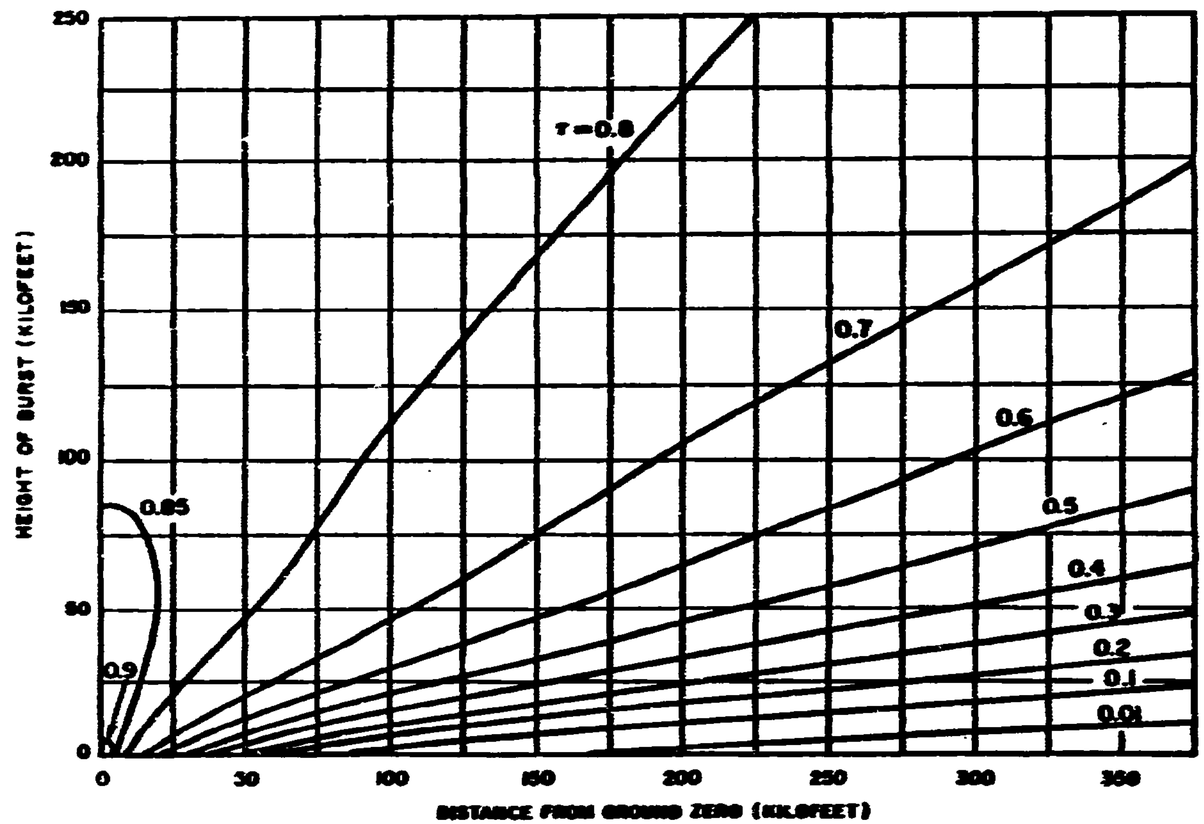


Fig. C-4. Transmittance to a Target on the Ground on a Typical Clear Day (Glasstone and Dolan, 1977:318)

2. The target is on the ground.

3. Visibility is 12 miles (21 km).

Figure C-4 is used to express τ as a function of distance for a surface burst by reading where the constant lines of τ intersect the bottom of the graph (zero altitude). Table C-I summarizes the values of τ taken from figure C-4.

TABLE C-I

VALUES OF τ AT HEIGHT OF BURST = 0
(Glasstone and Dolan, 1977:318)

τ	Distance D (kft)	Distance D (m)
0.01	168	51,206
0.1	85	25,908
0.2	58	17,678
0.3	50	15,240
0.4	38	11,582
0.5	31	9,449
0.6	20	6,096
0.7	15	4,572
0.8	9	2,743
0.9	5	1,524

Note: All distances have an uncertainty of ~2 kft.

When plotted as τ versus distance, the points in table C-I appear to follow an exponential curve that could be fit by an equation of the form:

$$\tau = \exp(f(D)) \tag{C.7}$$

where f(D) represents a function of distance D. By writing equation (C.7) as $\ln(\tau) = f(D)$, and using a linear regression program (Heilborn, 1981:16), the following expression was obtained:

$$f(D) = -.02455 - 6.439X10^{-5}D - 1.407X10^{-9}D^2 + 1.792X10^{-14}D^3 \qquad (C.8)$$

The points in table C-I and the curve produced by substituting equation (C.8) into equation (C.7) are shown in figure C-5. The coefficients in equation (C.8) were rounded to four significant digits, and the standard error estimate of f(D) (the standard deviation of ln(τ) about the fitted curve) is 0.05968. This error was considered to be well within the error inherent in reading from figure C-4.

Although the expression for τ was derived from data for a surface burst and ground target, the distance D in equation (C.8) is taken to be the slant range. This was done because slant range represents the distance in the atmosphere that the thermal radiation must travel to reach the missile. The expression for τ also does not consider the changing atmospheric conditions as the fireball rises or the effects of dust shielding the fireball. However, since τ is inherently uncertain because of its complex dependence on many variables, the expression derived for use in this analysis is considered to be adequate enough to show the change in transmittance as the missile moves away from the burst.

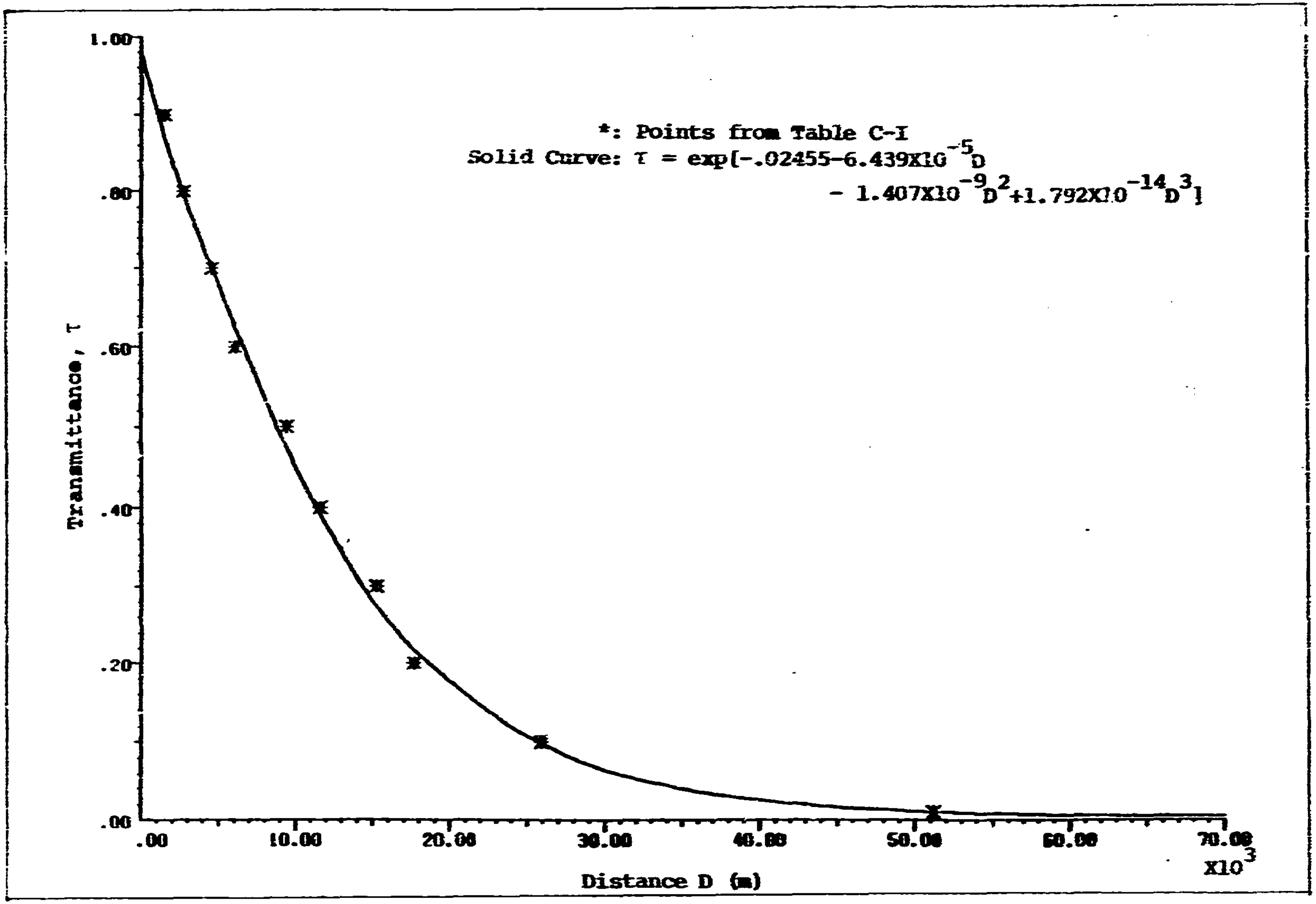


Fig. C-5. τ vs. Distance D

Appendix D. Ambient Air Properties

As mentioned in Chapter II, values for the ambient air temperature T_a, density ρ_a, dynamic viscosity μ_a, and conductivity k_a at the missile's altitude are needed to calculate the heat transfer coefficient and skin temperature. These values are obtained using equations given in the reference from the National Oceanic and Atmospheric Administration (NOAA, 1976:6-20).

The general equations for air temperature and pressure are:

$$\text{IF } L_k \neq 0: \quad T(z) = T_k + L_k(z - z_k)$$

$$P(z) = P_k \left[\frac{T_k}{T(z)}\right]^{(.034164/L_k)} \qquad \text{(D.1)}$$

$$\text{IF } L_k = 0: \quad T(z) = T_k$$

$$P(z) = P_k \exp\left[\frac{-.034164(z-z_k)}{T_k}\right] \qquad \text{(D.2)}$$

where z is the missile's altitude in meters, T(z) is the temperature in °K, and P(z) is the pressure in N/m^2. The values of L_k, T_k, and P_k depend on the region of the atmosphere and are given in table D-1.

TABLE D-I

VALUES USED TO CALCULATE AMBIENT AIR TEMPERATURE AND PRESSURE

Region	z_k (m)	L_k	T_k (°K)	P_k (N/m^2)
$0 \le z < 11{,}000$	0	-0.006545	288.15	1.013×10^5
$11{,}000 \le z < 20{,}000$	11,000	0.0	216.65	2.269×10^4
$20{,}000 \le z < 32{,}000$	20,000	0.0010	216.65	5.528×10^3
$32{,}000 \le z < 47{,}000$	32,000	0.0028	228.65	8.888×10^2

Once T(z) and P(z) are known, the remaining terms are calculated as follows:

$$\rho_a = .003484 \frac{P(z)}{T(z)} \quad kg/m^3 \tag{D.3}$$

$$\mu_a = \frac{1.458 \times 10^{-6}[T(z)]^{1.5}}{T(z)+110.4} \quad kg/m\text{-}s \tag{D.4}$$

$$k_a = \frac{2.64638 \times 10^{-3}[T(z)]^{1.5}}{T(z)+245.4[10^{(-12/T(z))}]} \quad J/m\text{-}s\text{-}°K \tag{D.5}$$

Appendix E. Local Leat Transfer Coefficient

The local convective heat transfer coefficient, h, was introduced in equation (2.14), which describes the rate of energy loss due to convection (Holman, 1976:12):

$$\dot{F}_{convection} = h[T(t)-T_{air}(t)] \tag{E.1}$$

where h has units of J/m^2-s-°K

The expressions for h used in this report were those derived for viscous flow over a flat plate. The derivation involved a detailed mathematical analysis of the thermal boundary layer, the region where temperature gradients are present in the flow along the plate (Holman, 1976:154-171). The results depend on the type of flow, which is described by the Reynolds number. For viscous flow, the Reynolds number is (Holman, 1976:149)

$$Re = \frac{vx\rho}{\mu} \tag{E.2}$$

where v = fluid free-stream velocity (m/s)

ρ = fluid density (kg/m^3)

μ = fluid dynamic viscosity (kg/m-s)

x = distance from leading edge of plate (m)

For air, $\rho = \rho_a$ and $\mu = \mu_a$ (see Appendix D). For this report, the velocity was taken to be the speed of the missile, and the value for $x = x_m$ was chosen to be 5.5 m, as explained at the end of this appendix. For flat plates, the transition between laminar and turbulent flow is considered to occur at Re = $5X10^5$ (Holman, 1976:149).

Another term used to calculate the heat transfer coefficient is the Prandtl number, which expresses the relative magnitudes of momentum and heat diffusion in the fluid. The expression for the Prandtl number is (Holman, 1976:164)

$$Pr = \frac{c_p \mu}{k} \tag{E.3}$$

where c_p = specific heat capacity of the fluid (J/kg-K)

K = thermal conductivity of the fluid (J/s-m-K)

For air, c_p = 1005 J/kg-K (Holman, 1976:503), $\mu = \mu_a$, and $k = k_a$ (see Appendix D).

From the analysis of the thermal boundary layer, the following expression is derived (Holman, 1976:170):

$$Nu = \frac{hx}{k} \tag{E.4}$$

so that

$$h = \frac{kNu}{x} \tag{E.5}$$

where Nu is the Nusselt number. For a plate heated over its entire length, the equations for Nu are if Re < $5X10^5$: $Nu = .322Pr^{1/3}Re^{1/2}$

(Holman, 1976:171) (E.6)

if Re > $5X10^5$: $Nu = .0296Pr^{1/3}Re^{4/5}$

(Holman, 1976:180) (E.7)

Thus, h can be calculated from equation (E.5) if the missile velocity and several air properties are known. As an example, at t = 5 seconds into the missile's flight, the missile velocity v = 140 ft/sec = 42.8 m/s and z = 600 ft = 182.9 m. The following values of ambient air properties at z = 182.9 m can be calculated from the equations in Appendix D:

$$\rho_a = 1.203 \text{ kg/m}^3$$

$$\mu_a = 1.784X10^{-5} \text{ kg/m-s}$$

$$k_a = .02523 \text{ J/m-s°K)}$$

Using these values in equations (E.2) and (E.3) results in

$$Re = \frac{(42.8)(5.5)(1.203)}{(1.784X10^{-5})} = 1.59X10^7$$

and

$$Pr = \frac{(1005)(1.784X10^{-5})}{(.02523)} = .711$$

Since Re > $5X10^5$, the flow is turbulent and the Nusselt number is given by equation (E.7):

$$Nu = .0296(.711)^{1/3}(1.59X10^7)^{4/5}$$
$$= 1.52X10^4$$

Substituting this value of Nu into equation (E.2) produces a value for the heat transfer coefficient h:

$$h = \frac{(.02523)(1.52X10^4)}{5.5} = 69.7\ J/m^2\text{-}s\text{-}°K$$

<u>Choosing a Value for x_m</u>

Figure E-1 shows the variation of h with Reynolds number for several values of x_m, where x_m is the distance from the leading edge of the missile. The data shows that the choice of x_m = 5.5 m represents an average between a maximum amount of heat transfer occurring for x_m = 1.0 m and a minimum amount of heat transfer occurring at x_m = 20 m.

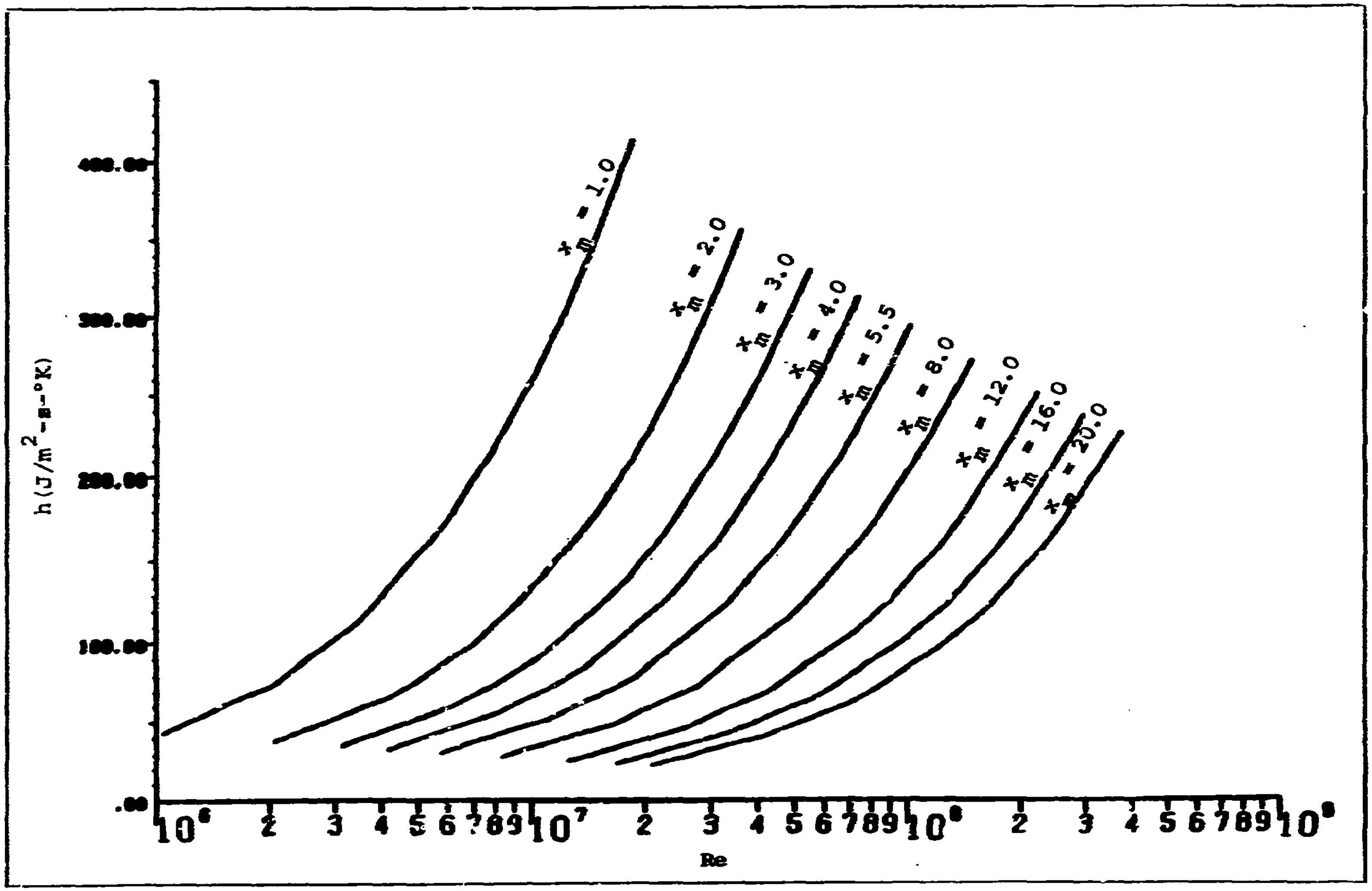


Fig. E-1. Variation of h with Reynolds Number for Various Values of x_m

Appendix F. Calculating the Fraction of Thermal Energy Emitted, ΔCT_j

The term CT refers to the curve shown in figure 4 in Chapter II. This curve represents the fraction of thermal energy emitted up to any time $t' = t/t_{max}$, where t is the time after detonation, $t_{max} = 0.417Y^{0.44}$ (Glasstone and Dolan, 1977:310), and Y is the yield in kilotons. At any time step j in the computer calculation of the missile skin temperature, the total thermal fluence that is incident on the surface during that time step is

$$\Delta Q_j = \Delta CT_j \cdot F_{incident} \qquad (F.1)$$

where $F_{incident}$ is given by equation (2.13). The curve in figure 4 must be digitized in order to calculate ΔCT_j during the run of the computer program. Sections of the curve were fit with second-degree equations or less using points from the curve and a linear regression program (Heilborn, 1981:16). Table F-I presents the points, equations, and correlation coefficients for the equations. Values of CT for $t' < 1.5$ were read directly from figure 4, while the points for $t' \geq 1.5$ were obtained with the help of a digitizer (owned by the Plasma Physics Group, Aero Propulsion Laboratory, Bldg 450, Wright-Patterson AFB, Ohio).

TABLE F-1

DATA AND EQUATIONS USED TO FIT FIGURE 4

Region	Values Used t'	Values Used CT	Equation (from Linear Regression)*	Correlation Coefficient**
$0 \le t' \le .75$	0	0	$CT = -.02t' + .24(t')^2$	1
	.25	.01		
	.50	.05		
	.75	.12		
$.75 < t' \le 1.5$	.75	.12	$CT = .32t' - .12$	1
	1.00	.20		
	1.25	.28		
	1.50	.36		
$1.5 < t' \le 2.5$+	1.50	.36	$CT = -.257219 + .556415t' - .0969029(t')^2$	.9998
	1.75	.4213		
	2.00	.4681		
	2.25	.5023		
	2.50	.5290		

*Linear regression program from Heilborn, with coefficients rounded to six figures where necessary.

**The correlation coefficient is a measure of how well the equation fits the given points. A perfect fit will have a correlation coefficient of 1.

+The values of CT used in this region (with the exception of $t'=10.0$) were obtained by linear interpolation from data presented in table F-2.

TABLE F-1--Continued

Region	Values Used t´	Values Used CT	Equation (from Linear Regression)*	Correlation Coefficient**
	2.50	.5290		
	3.00	.5752		
	3.50	.6125		
	4.00	.6459		
	4.50	.6711		
$2.5<t'<10.0$ +	5.00	.6936	$CT = .335808$	
	5.50	.7111	$+ .0949904t'$	
	6.00	.7272	$- .00494514(t')^2$	.9968
	6.50	.7391		
	7.00	.7530		
	7.50	.7650		
	8.00	.7734		
	8.50	.7822		
	9.00	.7907		
	10.00	.8000 ++		
$t'\geq 10.0$	-	.8000	$CT = .80$	-

++The value of CT = .80 at $t' = 10.0$ is taken directly from figure 4. All values of CT after $t' = 10.0$ are also taken as .80 since ΔCT between two values of t' greater than 10.0 is assumed to be negligible.

TABLE F-2

DATA FROM DIGITIZER

t'^{+}	CT^{++}
1.286297	0.295778
1.528967	0.367067
1.771637	0.426614
2.014308	0.470785
2.292048	0.507967
2.534717	0.532569
2.815257	0.560525
3.057937	0.579815
3.300607	0.598266
3.543277	0.615599
3.785947	0.632653
4.066487	0.649986
4.309158	0.659770
4.620568	0.678222
4.863238	0.687727
5.105907	0.698071
5.441158	0.709253
5.822697	0.721274
6.184597	0.733296
6.427267	0.736650
6.669937	0.744758
6.912607	0.750908
7.155278	0.756779
7.397948	0.762091
7.640628	0.769080
7.883297	0.770478
8.125967	0.776628
8.368637	0.780262
8.611307	0.783897
8.853978	0.789488
9.096648	0.791445

$$\left.\begin{array}{l} {}^{+}t' = t' * -.0168327 \\ {}^{++}CT = CT * + .167738 \end{array}\right\} \quad t' \text{ and CT normalized to 0,0.}$$

Given the equations in table F-I, the value of CT at any t' can be calculated to obtain ΔCT_j. As an example, for the second time step ($j=2$), the values of t' at the beginning and end of that time step for burst #1 are 1 and 2 respectively (see figure 7 in Chapter III). Using these values of t', ΔCT_2 is then:

$$\begin{aligned}\Delta CT_2 &= CT(t'=2) - CT(t'=1)\\ &= 0.4680 - 0.20\\ &= 0.268\end{aligned}$$

Appendix G. The Probability of Damage Function and CEP

The Probability Function

This appendix contains methods for calculating the probability of damage and quantifying the aiming error. The information was taken exclusively from class notes given by Dr. C. Bridgman (Bridgman, 1984).

For this report, probabilities of damage are calculated using intensity I as the independent variable and the cumulative log normal function as the distribution function. Each intensity has an associated probability of damage according to:

$$P_d(I) = \int_0^I \frac{1}{\sqrt{2\pi}\beta' I} \exp\left[-\frac{1}{2}\left(\frac{\ln I - \alpha'}{\beta'}\right)^2\right] dI \qquad (G.1)$$

where α' and β' are parameters that depend on the sure-safe and sure-kill intensities of the material. This integral can be solved analytically using the following substitution:

$$u = \frac{\ln I - \alpha'}{\beta'} \qquad (G.2)$$

Substituting equation (G.2) into the integral of equation (G.1) produces

$$P_d(u) = \int_{-\infty}^{u} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du \qquad (G.3)$$

whose integrand is recognized as the normal distribution function about $u = 0$. A graph of the above integral is shown in figure G-1.

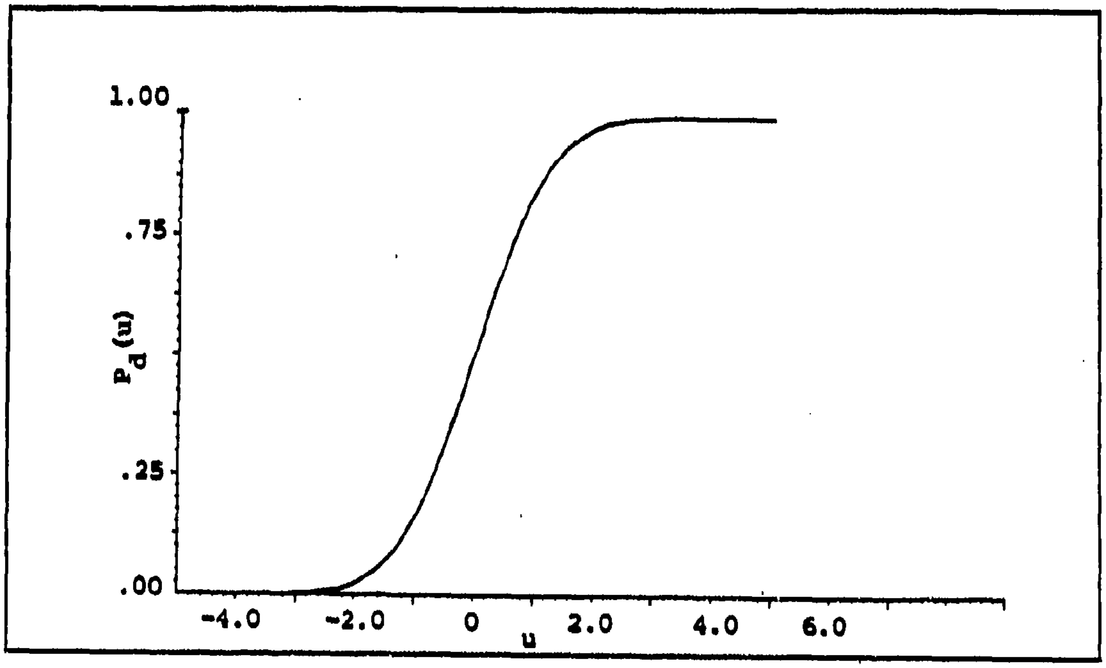


Fig. G-1. Cumulative Normal Distribution Function

Equation (G.3) can be solved in any number of ways. A good method for computer implementation is to use the following equations (Abramowitz and Stegun, 1965:932):

$$u' = |u|$$

$$P(u') = 1 - \frac{1}{[2(1+.196854u'+.11594(u')^2+.000344(u')^3+.019527(u')^4]^4} \qquad (G.4)$$

If $u \geq 0$, $P_d(I) = P(u')$

If $u < 0$, $P_d(I) = 1 - P(u')$

Thus, once an intensity I is found and α' and β' are known, $P_d(I)$ can be calculated using equations (G.2) and (G.4) or equation (G.2) and the curve in figure G-1.

As mentioned before, the parameters α' and β' are calculated from sure-safe and sure-kill intensities I_{ss} and I_{sk}. For this analysis, all probabilities of damage below 0.02 are considered to be sure-safe ($P_d(I)=0$), and all probabilities above 0.98 are considered to be sure-kill ($P_d(I)=1.0$). Thus, α' and β' must be defined so that if $P_d(I_{sk}) = 0.98$ and $P_d(I_{ss}) = 0.02$ then the following are true:

$$.98 = \int_0^{I_{sk}} \frac{1}{\sqrt{2\pi}\,\beta' I} \exp\left[-\frac{1}{2}\left(\frac{\ln I - \alpha'}{\beta'}\right)^2\right] dI \qquad (G.5)$$

$$.02 = \int_0^{I_{ss}} \frac{1}{\sqrt{2\pi}\,\beta' I} \exp\left[-\frac{1}{2}\left(\frac{\ln I - \alpha'}{\beta'}\right)^2\right] dI \qquad (G.6)$$

The solution to the above equations yields

$$\frac{\ln I_{ss} - \alpha'}{\beta'} = -2.054 \qquad (G.7)$$

$$\frac{\ln I_{sk} - \alpha'}{\beta'} = 2.054 \qquad (G.8)$$

which can in turn be solved to give

$$\alpha' = \frac{1}{2}\ln(I_{ss} \cdot I_{sk}) \tag{G.9}$$

$$\beta' = \frac{1}{4.108}\ln\left(\frac{I_{sk}}{I_{ss}}\right) \tag{G.10}$$

The expressions for α' and β' given by equations (G.9) and (G.10) are used to calculate u in equation (G.2)

Aiming Error or CEP

The development of the function describing the aiming error of an RV around designated ground zero (DGZ) assumes the following:

1. Errors are in the x and y directions only. The height of burst (z=0) is accurate.

2. The errors in x and y are entirely random.

3. $\sigma_x = \sigma_y$.

If f(x) = f(y) are functions describing the errors in x and y respectively, then

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma_x}\right)^2\right] \tag{G.11}$$

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma_y} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \tag{G.12}$$

These two distribution functions are shown in figure G-2 as two gaussians centered at the aim point (DGZ) and extending to define a circular area A. The probability of hitting area A is

$$f(x,y) = f(x)f(y) = \frac{1}{2\pi\sigma^2}\exp[-\frac{1}{2}(\frac{x^2+y^2}{\sigma^2})] \qquad (G.13)$$

which is a circular area density or planar normal function.

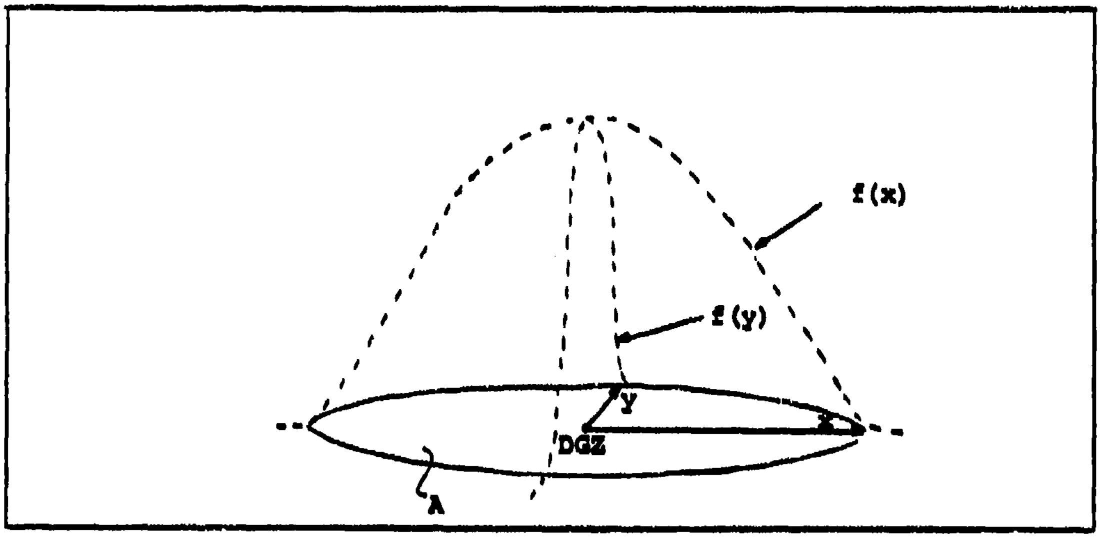


Fig. G-2. Planar Normal Function

Equation (G.13) can be written for a differential area dA as

$$f(r)dA = \frac{1}{2\pi\sigma^2}\exp[-\frac{1}{2}(\frac{r}{\sigma})^2]\ 2\pi r dr \qquad (G.14)$$

where $r = x^2 + y^2$, the radius of the circle defined by f(x,y). The probability of falling anywhere inside of radius r is then

$$F(r) = \int_0^r f(r)dA = \int_0^r \frac{1}{2\pi\sigma^2}\exp[-\frac{1}{2}(\frac{r}{\sigma})^2\ 2\pi r dr \qquad (G.15)$$

F(r) is a cumulative circular normal function. By substituting $r = s\sigma$ and recognizing that $f(s) = f(r)(dr/ds)$, the above equation can be written as

$$F(s) = \int_0^{s\sigma} \exp[-\tfrac{1}{2}s^2]\ s\,ds \qquad (G.16)$$

F(r) can be evaluated for different values of s as follows:

s	F
1	0.393
2	0.865
3	0.989

The term circular error probable, or CEP, is defined as the radius r inside of which 50% of the targeted RV's will hit. According to the above values of F, this radius is between σ and 2σ. Specifically, using equation (G.15)

$$.50 = \int_0^{CEP} \frac{1}{2\pi\sigma^2} \exp[-\tfrac{1}{2}(\tfrac{r}{\sigma})^2]\ 2\pi r\,dr \qquad (G.17)$$

By substituting $w = 0.5(r/\sigma)^2$, the above integral can be solved analytically to obtain

$$CEP = \sqrt{2 \ln 2} \simeq 1.18\sigma \qquad (G.18)$$

This result will be used to define areas of equal probability around the designated ground zero.

Combining Probability of Damage and Aiming Error

The overall probability of damage to a missile depends on the following:

a. the probability of the RV hitting any dA surrounding the DGZ

b. the probability of damage given a hit on any dA

The condition in (a) is given by equation (G.15). Rewriting equation (G.15) in polar coordinates, where $r = \rho$ and $dA = d\rho d\theta$, yields:

$$f(p)dA = \frac{1}{2\pi\sigma^2} \exp[-\frac{1}{2}(\frac{\rho}{\sigma})^2]\ \rho d\rho d\theta \qquad (G.19)$$

The condition in (b) is given by $P_d(I)$ (equation (G.1)). Thus, the probability of damage over all dA is

$$P_d = \int_0^\infty \int_0^{2\pi} f(\rho)\ P_d(I)dA$$

$$= \int_0^\infty \int_0^{2\pi} \left\{ \frac{1}{2\pi\sigma^2} \exp[-\frac{1}{2}(\frac{\rho}{\sigma})^2] \int_0^I \frac{1}{\sqrt{2\pi}\beta' I} \exp[-\frac{1}{2}(\frac{\ln I - \alpha'}{\beta'})^2 dI \right\} \rho d\rho d\theta \qquad (G.20)$$

This integral can be simplified to a sum over N areas, designated as $\Delta_{\text{area } i}$, as follows:

$$P_d = \sum_{i=1}^{N} f(\rho)_i\ P_{d_i}(I)\Delta_{\text{area } i} \qquad (G.21)$$

The areas are now chosen so that

$$f(\rho)_i \Delta_{\text{area } i} = \frac{1}{N} \qquad \text{(G.22)}$$

That is, the space around designated ground zero is divided into areas, or cells, of equal probability of getting hit. Thus, the probability of damage for a missile becomes

$$P_d = \frac{1}{N} \sum_{i=1}^{N} P_{d_i}(I) \qquad \text{(G.23)}$$

The probability of damage from a hit in cell i, $P_{d_i}(I)$, is calculated by finding the intensity I at the slant range determined from the centroid of cell i (see Appendix C), and then using equations (G.2) and (G.4). The $P_{d_i}(I)$'s found for each cell are summed and divided by the total number of cells, N, to obtain P_d. Knowing P_d, the probability of survival $P_s = 1 - P_d$.

<u>Calculating the Cell Centroids</u>

A 10-cell space around DGZ is shown in figure G-3. For convenience, each cell is numbered. The dot inside each cell is the cell centroid, whose position must be known to calculate the slant range. Each centroid is defined by $\langle\rho_i\rangle$ and $\langle\theta_i\rangle$, where $\langle\rho_i\rangle$ is the distance to the centroid of cell i from DGZ (the center of cell #1), and $\langle\theta_i\rangle$ is the angle between $\langle\rho_i\rangle$ and the x-axis. $\langle\rho_i\rangle$

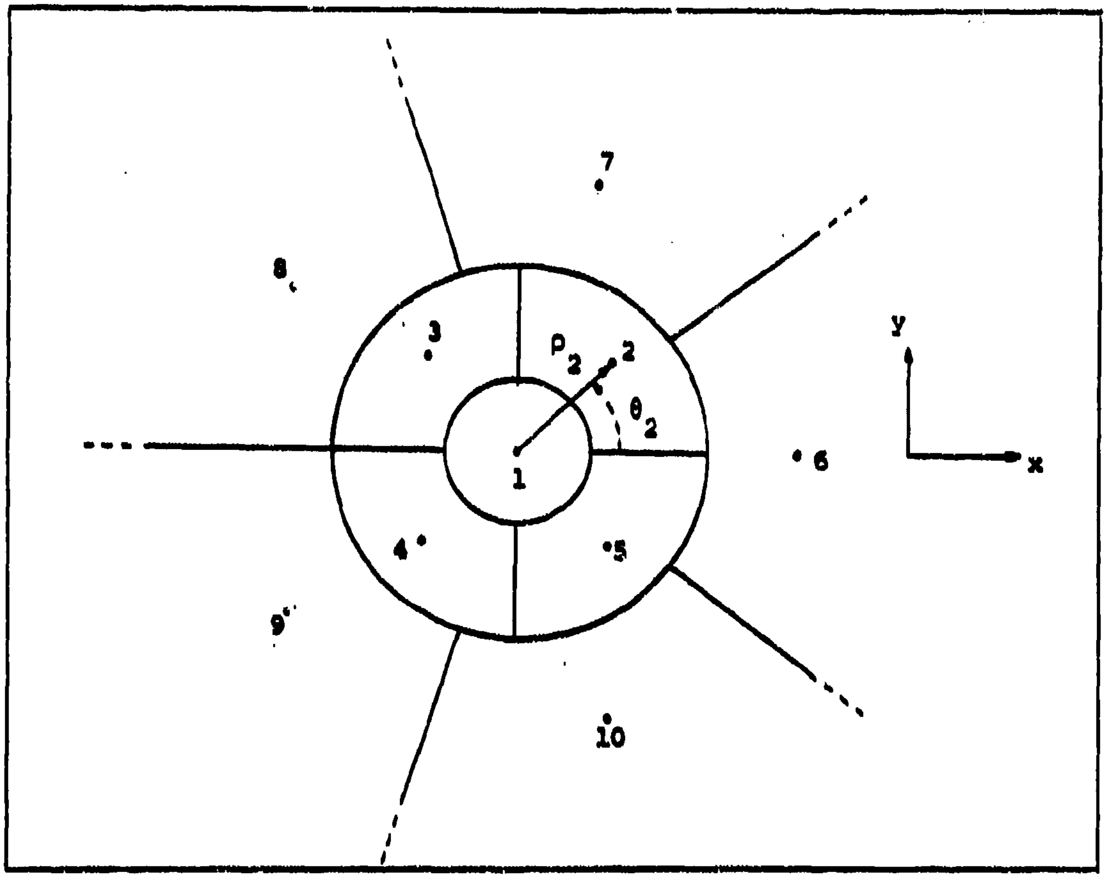


Fig. G-3. A 10-cell Discretized Space Around DGZ

and $<\theta_i>$ are derived such that they define cells of equal probability.

A number of steps are necessary to obtain an expression for $<\rho_i>$ using the following variables:

ρ_i: outer radius of cell i

n_r: number of cells in ring r

N_r: number of cells inside ring r

N_T: total number of cells

A ring of cells is a group of cells with equal radii, such as cells #2, 3, 4, and 5. Therefore, for any cell in ring r_i, ρ_i of that cell is synonymous to ρ_r for the ring. For a 10-cell configuration, the following table can be constructed:

Ring	n_r	N_r
1	1	1
2	4	5
3	5	10

Since equal probability cells are desired, the total probability of hitting inside ring r is N_r/N_T, and in turn this is equal to

$$\frac{N_r}{N_T} = \int_0^{\rho_r} \frac{1}{2\pi\sigma^2} \exp[-\frac{1}{2}(\frac{\rho}{\sigma})^2]\ 2\pi\rho d\rho d\theta \qquad (G.24)$$

for all θ. The above integral can be solved analytically using the substitution $w = \rho/\sigma$ to obtain

$$\frac{N_r}{N_T} = 1 - \exp[-\frac{1}{2}(\frac{\rho_r}{\sigma})^2] \qquad (G.25)$$

Solving equation (G.25) for ρ_r results in

$$\rho_r = \sigma[-2 \ln (1 - \frac{N_r}{N_T})]^{\frac{1}{2}} \qquad (G.26)$$

Equation (G.26) can be generalized by expressing σ in terms of CEP using equation (G.13):

$$\frac{\rho_r}{CEP} = \left[\frac{-\ln(1-\frac{N_r}{N_T})}{\ln 2}\right] \tag{G.27}$$

The following equation is used to find $\langle\rho_r\rangle$:

$$\langle\rho_r\rangle = \frac{\int_{\rho_{r-1}}^{\rho_r} \rho f(\rho)\ 2\pi\rho d\rho}{\int_{\rho_{r-1}}^{\rho_r} f(\rho) 2\pi\rho d\rho} \tag{G.28}$$

Substituting the expression for $f(\rho)$, and using equation (G.24) yields

$$\langle\rho_r\rangle = \frac{N_T}{N_r}\sigma \int_{\rho_{r-1}}^{\rho_r} \exp[-\frac{1}{2}(\frac{\rho}{\sigma})^2]\ \frac{\rho^2}{\sigma^2}\, d\rho \tag{G.29}$$

Equation (G.29) is solved by making the substitution $w = \rho/\sigma$ and then integrating by parts. In terms of CEP, the result is

$$\frac{\langle\rho_r\rangle}{CEP} = \frac{N_T}{N_r}\left\{ \frac{\rho_{r-1}}{CEP} \exp[-\ln 2(\frac{\rho_{r-1}}{CEP})^2] - \frac{\rho_r}{CEP} \exp[-\ln 2(\frac{\rho_r}{CEP})^2] \right.$$

$$\left. + (\frac{\pi}{\ln 2})^{.5} [CNF((2\ln 2)^{.5}\frac{\rho_r}{CEP}) - CNF((2\ln 2)^{.5}\frac{\rho_{r-1}}{CEP})]\right\} \tag{G.30}$$

where CNF(x) = cumulative log normal function for an argument x, as evaluated in equation (G.4). For the 10-cell system, the following values are obtained from equations (G.27) - (G.30):

TABLE G-I

VALUES OF $\langle\rho_i\rangle$

Ring r	ρ_i/CEP	$\langle\rho_i$/CEP$\rangle$
1	0.3899	0.0
2	1.0	0.7109
3	∞	1.509

Thus, $\langle\rho_r\rangle$ can be found by multiplying $\langle\rho_r$/CEP$\rangle$ by the given value of CEP.

Table G-II shows values of θ_i found by inspection of figure G-3.

TABLE G-II

VALUES OF $\langle\theta_i\rangle$

Cell #i	θ_i
1	0
2	0.25π
3	0.75π
4	1.25π
5	1.75π
6	0
7	0.4π
8	0.8π
9	1.2π
10	1.6π

Appendix H. Data for Choosing the Upper Limit of Bursts

As mentioned in Chapter III, the upper limit to the number of bursts that will cause a missile's skin temperature to rise was determined to be eight. This limit was estimated by calculating the maximum temperature reached for a series of bursts and assuming that each RV lands at its designated ground zero (a 1-cell CEP). The following table summarizes the data used to arrive at the upper limit by including the results for 4, 7, 8 and 9 bursts. It should be noted that missiles below #13 required fewer than eight bursts to reach a maximum temperature, but these missiles have no probability of survival for even four bursts.

TABLE H-1

DATA USED TO DETERMINE THE UPPER LIMIT OF BURSTS FOR A 1-CELL CEP

Missile #	Maximum Temperature Reached (°K)* 4 Bursts j=8	7 Bursts j=12	8 Bursts j=13	9 Bursts j=13
21	2,615	3,602	3,611	3,611
23	2,576	3,543	3,564	3,564
31	1,175	1,550	1,565	1,565
33	1,169	1,541	1,558	1,558
37	891	1,145	1,157	1,157
41	710	886	895	895
43	708	883	893	893
47	594	721	729	729
51	514	607	612	612
53	513	606	611	611
57	458	528	532	532
61	418	471	474	474
63	418	470	473	473
67	389	429	431	431
71	367	398	399	399
73	366	397	399	399
77	350	374	375	375
81	337	356	357	357
83	337	356	357	357
93	320	331	331	331

*Temperatures are rounded to the nearest degree; however, all maximum temperatures for 8 and 9 bursts were equal before rounding.

j = timestep when maximum temperature was reached.

Fireball rise considered for all results.

Missiles were launched when silo #1 was hit (tb_1 = 0 sec).

Appendix I. Data for Choosing the Maximum Number of Time Steps

As mentioned in Chapter III, the maximum number of time steps needed to determine the maximum skin temperature was chosen to be 11. This choice was based on the results from a 1-cell CEP, 4 burst scenario for several missiles. Table I-1 summarizes the results for eight missiles along the length of the field.

TABLE I-1

DATA USED TO DETERMINE MAXIMUM NUMBER OF NECESSARY TIME STEPS (1-CELL CEP)

Time Step \ Missile #	Skin Temperature (°K) at Each Time Step							
	21	31	41	51	61	71	81	91
1	495.26	366.60	325.38	308.09	299.65	295.12	292.53	290.98
2	781.65	475.81	377.35	335.94	315.72	304.87	298.65	294.93
3	1159.99	621.34	446.83	373.25	337.27	317.94	306.87	300.23
4	1475.01	743.91	505.59	404.86	355.54	329.03	313.84	304.72
5	1835.95	887.43	575.00	442.36	377.27	342.22	322.13	310.07
6	2204.20	1025.91	640.35	477.19	397.26	354.28	329.66	314.89
7	2541.00	1148.15	697.09	507.11	414.31	364.49	335.99	318.91
8	2615.06	1175.23	709.62	513.65	417.95	366.60	337.22	319.62
9	2571.97	1159.16	701.93	509.42	415.39	364.93	336.05	318.75

Fireball rise considered for all results.

Missiles were launched when silo #1 was hit ($tb_1 = 0$ sec).

Appendix J. Data for Figures in Chapter IV

This appendix contains tables of data that were used to plot figures 8, 9, and 10.

TABLE J-I

DATA USED TO PLOT FIGURE 8, TEMPERATURE RISE FOR THE CUMULATIVE CASE (1-CELL CEP)

1 Burst		2 Bursts		3 Bursts		4 Bursts	
j	T_2(°K)	j	T_2(°K)	j	T_2(°K)	j	T_2(°K)
1	325.40	1	325.40	1	325.40	1	325.40
2	374.45	2	377.49	2	377.49	2	377.49
3	392.67	3	447.08	3	447.08	3	447.08
4	400.78	4	491.91	4	505.69	4	505.69
5	405.62	5	508.35	5	574.46	5	574.46
6	406.99	6	514.60	6	604.06	6	638.73
7	404.98	7	513.99	7	608.34	7	693.58
8	400.28	8	507.85	8	604.07	8	704.54
9	393.33	9	496.97	9	592.34	9	695.66
10	385.51	10	483.00	10	575.02	10	677.84
		11	466.01	11	552.79	11	652.34
				12	527.99	12	622.02
				13	502.76	13	589.68

Missile #41.
Launch time: 0 sec.
Time of first burst: 0 sec.
Fireball rise not considered.

TABLE J-II

DATA USED TO PLOT FIGURE 9

Time step = t_{max}		Time step = $t_{max}/2$		
j	T_2(°K)	j	t/t_{max}	T_2(°K)
1	325.38	1	0.25	297.48
2	377.35	2	0.75	325.37
3	446.83	3	1.25	354.82
4	505.59	4	1.75	377.30
5	575.00	5	2.25	409.94
6	640.35	6	2.75	446.75
7	697.09	7	3.25	475.00
8	709.62	8	3.75	505.45
9	701.93	9	4.25	543.49
10	685.22	10	4.75	574.80
11	660.33	11	5.25	601.23
12	630.17	12	5.75	640.16
13	597.60	13	6.25	674.26
		14	6.75	696.77
		15	7.25	706.13
		16	7.75	709.25
		17	8.25	706.72
		18	8.75	701.46
		19	9.25	693.58
		20	9.75	684.81
		21	10.25	673.15
		22	10.75	659.95
		23	11.25	645.37
		24	11.75	629.86
		25	12.25	613.80
		26	12.75	597.31
		27	13.25	580.40
		28	13.75	563.43
		29	14.25	546.58
		30	14.75	529.71

Missile #41.
Launch time: 0 sec.
Time of first burst: 0 sec.
Fireball rise considered.

TABLE J-III

DATA USED TO PLOT FIGURE 10
(Cumulative Results Given in Table J-II)
(Noncumulative Results)

Burst #1		Burst #2		Burst #3		Burst #4	
j	T_2 (°K)	j	T_2 (°K)	j	T_2 (°K)	j	T_2 (°K)
1	325.38	1	324.33	1	321.99	1	326.51
2	374.31	2	371.75	2	366.19	3	376.54
3	392.41	3	388.05	3	379.86	3	390.88
4	400.50	4	394.30	4	383.78	4	393.82
5	405.42	5	396.80	5	384.74	5	393.34
6	406.96	6	396.31	6	382.91	6	389.89
7	405.14	7	393.12	7	378.67	7	383.91
8	400.62	8	387.40	8	372.42	8	376.05
9	393.78	9	379.77	9	364.52	9	366.51
10	386.06	10	371.68	10	356.54	10	357.00

Missile #41.
Launch time: 0 sec.
Time of first burst: 0 sec.
Fireball rise considered.

Appendix K. Computer Programs

This appendix contains two computer programs. The first one presented is Program Therm, written in FORTRAN77. This program calculates the probability of survival for the cumulative case. Preceding the program is a list of variables and a flow chart. The second program, written in Basic, calculates the probability of survival in the noncumulative case.

Variable Listing for Program THERM

NAME	UNITS	DESCRIPTION
Main Program:		
a	J/m^2-°K	$a = c_p \rho d$
alfa	-	absorptivity of aluminum
alpha	-	$\alpha' = .5*\ln(I_{ss}*I_{sk})$
alt(11)	-	array containing altitude at time step j
ang(11)	radians	array containing missile flight path angle at time step j
angdata(71)	degrees	array containing data for missile flight path angle
B1,B2,B3,B4	-	cell number of burst 1,2,3,4
beta	-	$\beta' = (1/4.108)*\ln(I_{sk}/I_{ss})$
Cp	J/Kg-°K	specific heat capacity
d	m	thickness of missile skin
dCT(4,11)	-	array containing ΔCT for burst k at time step j
drd	m	missile down-range-distance from first row of silos
h(11)	J/m^2-s-°K	array containing heat transfer coefficient at time step j
Hfb(4,11)	m	array containing height of fireball for burst k at midpoint of time step j
HiT	°K	highest maximum temperature reached during calculation of P_s

NAME	UNITS	DESCRIPTION
I_{sk}	°K	sure-kill intensity
I_{ss}	°K	sure-safe intensity
j	-	time step
k	-	burst #
LowT	°K	lowest maximum temperature reached during calculation of P_s
lt	s	missile launch time
maxb	-	maximum number of bursts ($\leq$ 4)
Mn	-	missile #
Ps	-	probability of survival
rho	kg/m^3	density of aluminum
rhocep(10)	-	array containing values of the distance to the centroid of cell i
sumPd	-	sum of P_d(MaxT) for each burst-missile configuration
sx(100)	m	x-coordinate of silo position
sy(100)	m	y-coordinate of silo position
Ta0	°K	ambient air temperature at time of first burst
tb1	s	time of first burst
Temp(11)	°K	array containing ambient air temperature at time step j
tf	-	thermal fraction
theta(10)	radians	array containing values of the angular location of the centroid cell of i

NAME	UNITS	DESCRIPTION
ul	-	upper limit to # of cells considered (≤ 10)
vdata(51)	ft/s	array containing data for missile velocity as a function of time
vel(11)	m/s	array containing data for missile velocity at time step j
W	MT	yield in megatons
xdata(51)	ft	array containing data for missile altitude as a function of time
zdata(51)	ft	array containing data for missile distance from silo as a function of time

Subroutines: Only ambiguous variables are listed

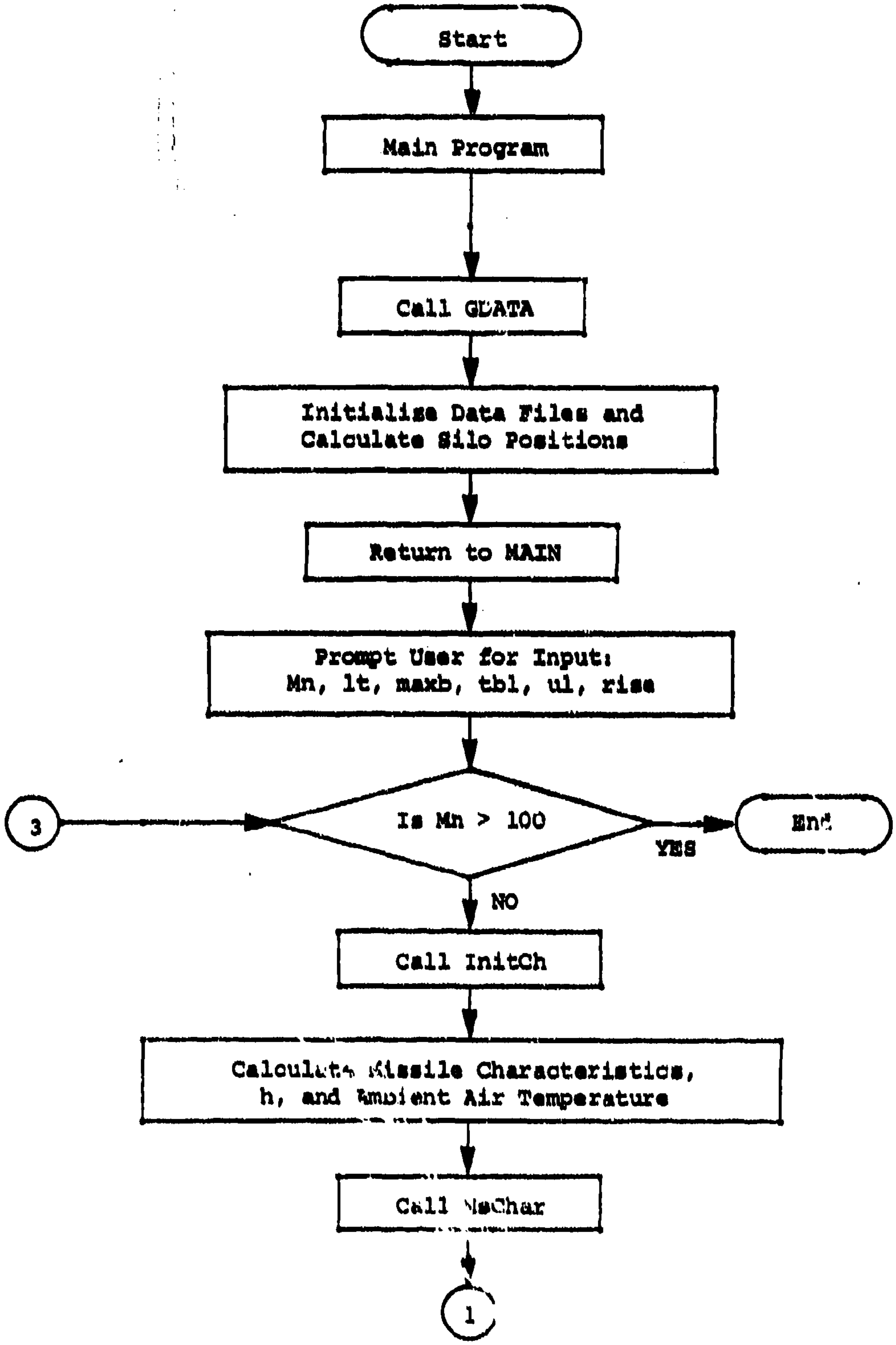
TCalc:

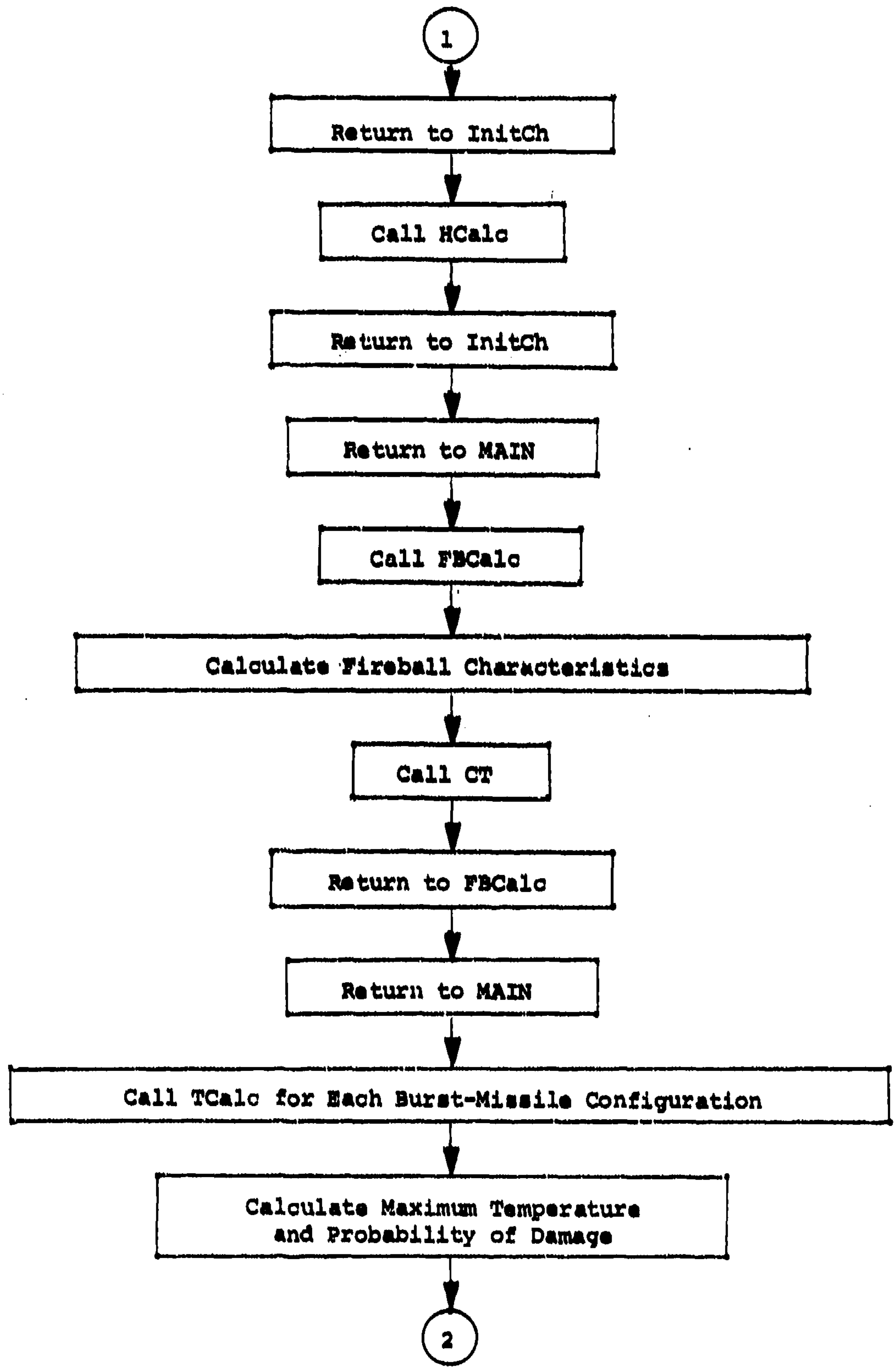
nb	-	number of bursts
tb	s	time of burst
sb	-	silo of burst
newbst	-	logical variable: TRUE when another burst occurs during j
z	-	$z = (\ln MaxT - \alpha')/\beta'$

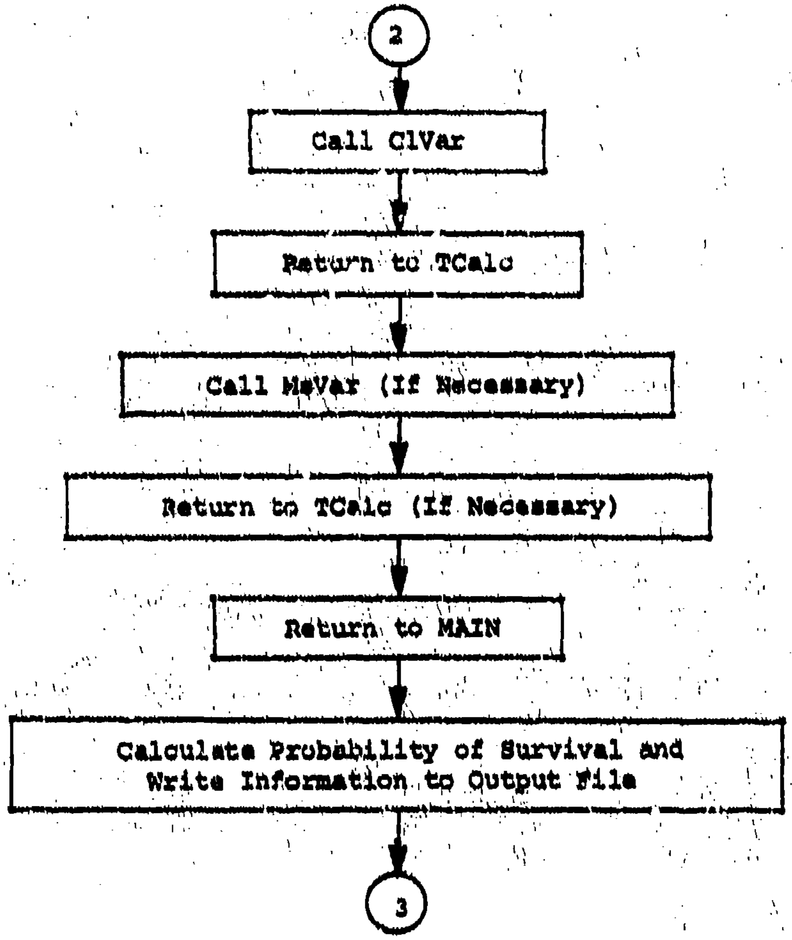
ClVar:

dgzx,dgzy	m	x and y coordinate of designated ground zero
Mny	m	missile y coordinate

NAME	UNITS	DESCRIPTION
InitCh:		
tm	s	time into missile flight
FBCalc:		
tb	s	time of burst
tp	-	normalized time
t	s	time after burst
CTd,CTu	-	fraction of thermal energy emitted up to beginning and up to end of time step j
MsChar:		
Mnx	m	missile x coordinate
td,tu	s	integer times above and below t
t	s	time of interest
HCalc:		
cp	J/kg-°K	specific heat capacity of air
Re	-	Reynolds number
Pr	-	Prandtl number
Nu	-	Nusselt number
Ta	°K	ambient air temperature
P	N/m^2	ambient air pressure
rhoa	kg/m^3	ambient air density
mu	kg/m-s	ambient air viscosity
kappa	J/m-s-°K	ambient air conductivity
xm	m	point along missile skin


Start
Main Program
Call GDATA
Initialize Data Files and
Calculate Silo Positions
Return to MAIN
Prompt User for Input:
Mn, lt, maxb, tbl, ul, rise
3
Is Mn > 100
YES
End
NO
Call InitCh
Calculate Missile Characteristics,
h, and Ambient Air Temperature
Call MsChar
1


1
Return to InitCh
Call HCalc
Return to InitCh
Return to MAIN
Call FBCalc
Calculate Fireball Characteristics
Call CT
Return to FBCalc
Return to MAIN
Call TCalc for Each Burst-Missile Configuration
Calculate Maximum Temperature
and Probability of Damage
2


2
Call ClVar
Return to TCalc
Call MsVar (If Necessary)
Return to TCalc (If Necessary)
Return to MAIN
Calculate Probability of Survival and
Write Information to Output File
3

```
*******************************************************************
*                                                                 *
*                        PROGRAM Therm                            *
*                                                                 *
*          Written by 2Lt Barbara A. Hall, GNE-85M                *
*              Air Force Institute of Technology                  *
*                     Fall Quarter, 1984                          *
*                                                                 *
*  This program calculates the probability of survival of a missile *
*  subjected to thermal radiation from a series of bursts for the *
*  cumulative case.  The scenario modeled in this program is the  *
*  Peacekeeper close-spaced basing formation, or Dense Pack.      *
*  Details of the scenario are as follows:                        *
*                                                                 *
*  Threat : Walk attack starting at silo #1 and proceeding every 2 *
*          seconds to successive silos                            *
*          Surface bursts                                         *
*          Yield of each burst : 2 MT                             *
*          Aiming error only for designated ground zero           *
*                                                                 *
*  Missile : Aluminum skin                                        *
*          Iss = 619 K, Isk = 809 K                               *
*                                                                 *
*  The program will prompt the user for the following information: *
*                                                                 *
*      Missile #                                                  *
*      Launch time (sec)                                          *
*      Number of bursts to be considered : must be <= 4           *
*      Time of first burst (sec) : must be an even number and     *
*                                  >= to the launch time          *
*      Number of cells to be considered : between 1 and 10        *
*      Option for fireball rise : rise = 1                        *
*                                                                 *
*  Results of the calculation will be placed in the file "thm.dat" *
*  Thm.dat will contain the above information plus the lowest and *
*  highest maximum temperatures calculated during the run and the *
*  missile's probability of survival.                             *
*                                                                 *
*******************************************************************

      PROGRAM Therm

      INTEGER tb1,Mn,maxb,B1,B2,B3,B4,ul,rise
      REAL Iss,Isk,Y,W,alpha,beta,tmax,tf,rho,alfa,Cp,d,a,lt
      REAL vdata,zdata,xdata,angdat,sx,sy,rhocep(10),theta(10)
      REAL vel,alt,drd,ang,h,Temp,Ts0,dCT,Hfb,LowT,HiT,sumPd,Ps
      COMMON /block1/vdata(51),zdata(51),xdata(51),angdat(71)
      COMMON /block2/sx(100),sy(100)
      COMMON /block3/vel(11),alt(11),drd(11),ang(11),h(11),Temp(11)
      COMMON /block4/dCT(4,11),Hfb(4,11)
      COMMON /blockd/tf,alfa,Y,a,alpha,beta
      DATA B1,B2,B3,B4 /4*0/
```

```
      DATA rhocep /0.0,.7109,.7109,.7109,.7109,1.509,1.509,1.509,
     +            1.509,1.509/
      DATA theta /0.0,.78539,2.35619,3.92698,5.49778,0.0,1.25663,
     +           2.51327,3.76991,5.02654/

C  Main Program
      Iss = 819.
      Isk = 809.
      Y = 2000.
      alpha = (1./2.)*ALOG(Iss*Isk)
      beta = (1./4.108)*ALOG(Isk/Iss)
      tmax = .0417*(Y**.44)
      tf = .18
C     Values for Aluminum:
      rho = 2700.
      alfa = .50
      Cp = 900.
      d = .001
      a = Cv*rho*d
      CALL Gdata
      OPEN(2,file='thm.dat',status='new',access='sequential',form=
     +     'formatted')

    1 WRITE (*,'(A±)') ' Missile # (Enter 101or greater to exit): '
      READ (*,'(BN,I4)') Mn
      IF (Mn .GT. 101) GOTO 100
      WRITE (*,'(A±)') ' Designated launch time (sec) :'
      READ (*,'(BN,F4.1)') lt
    2 WRITE (*,'(A±)') ' Number of bursts  (<=4) to be considered: '
      READ (*,'(BN,I1)') maxb
      IF (maxb .GT. 4) GOTO 2
    3 WRITE (*,'(A±)') ' Time of first burst (must be >= lt & even): '
      READ (*,'(BN,I3)') tb1
      IF (tb1 .LT. lt .OR. INT(tb1/2) .NE. tb1/2) GOTO 3
    4 WRITE (*,'(A±)') ' Number of cells to be considered: '
      READ (*,'(BN,I3)') ul
      IF (ul .LT. 1 .OR. .GT. 10) GOTO 4
    5 WRITE(*,'(A±)') ' Type 1 if want fireball rise, 0 if not: '
      READ(*,'(BN,I2)') rise
      IF (rise .NE. 0 .OR. rise .NE. 1) GOTO 5

      CALL InitCh(tb1,lt,tmax,Mn,Ta0)
      W = Y/1000.
      CALL FBCalc(maxb,tb1,tmax,rise,W)
C     LowT  and HiT are initialized
      LowT = 1.0e+8
      HiT = 0.0
      sumPd = 0.0
C     Calculate the maximum skin temperature and probability of damage
C     for a total of 10**maxb burst - missile configurations.
C     Rk = current cell number of burst k
      IF (maxb .EQ. 1) THEN
```

```
         DO 10 B1 = 1,ul
            CALL TCalc(B1,B2,B3,B4,Mn,tmax,tb1,maxb,TaO,rhocep,theta,
     +                sumPd,LowT,HiT,lt)
  10     CONTINUE
      ELSE IF (maxb .EQ. 2) THEN
         DO 20 B2 = 1,ul
          DO 20 B1 = 1,ul
            CALL TCalc(B1,B2,B3,B4,Mn,tmax,tb1,maxb,TaO,rhocep,theta,
     +                sumPd,LowT,HiT,lt)
  20     CONTINUE
      ELSE IF (maxb .EQ. 3) THEN
         DO 30 B3 = 1,ul
          DO 30 B2 = 1,ul
           DO 30 B1 = 1,ul
            CALL TCalc(B1,B2,B3,B4,Mn,tmax,tb1,maxb,TaO,rhocep,theta,
     +                sumPd,LowT,HiT,lt)
  30     CONTINUE
      ELSE
         DO 40 B4 = 1,ul
          DO 40 B3 = 1,ul
           DO 40 B2 = 1,ul
            DO 40 B1 = 1,ul
            CALL TCalc(B1,B2,B3,B4,Mn,tmax,tb1,maxb,TaO,rhocep,theta,
     +                sumPd,LowT,HiT,lt)
  40     CONTINUE
      END IF

C  Write to file
      WRITE(2,89)
  89  FORMAT('    ')
      WRITE(2,90)Mn,lt,maxb,tb1,ul
  90  FORMAT(' MX #',I2,'  launch t = ',F5.2,'  maxb=',I2,'  tb1='
     +       ,I2,'  # cells : ',I2)
      IF (rise .EQ. 1) WRITE(2,91)
  91     FORMAT('    Fireball rise considered')
      WRITE(2,97)LowT,HiT
  97  FORMAT('   Lowest T = ',F9.3,'    Highest T = ',F9.3)
      Ps = 1.0 - (sumPd/(ul**maxb))
      WRITE(2,95)Ps
  95  FORMAT(' Ps = ',F8.4)
      GOTO 1

 100  CLOSE (2)
      END
```

```
**********************************************************************
*                                                                    *
*          SUBROUTINE TCalc - Temperature Calculation                *
*                                                                    *
*  Passed : B1, B2, B3, B4, Mn, tmax, tb1, maxb, Ta0, rhocep(10),    *
*          theta (10), sumPd, LowT, HiT, lt                          *
*  COMMON : ax(100), ay(100), vel(11), alt(11), drd(11), ang(11),    *
*          h(11), Temp(11), dCT(4,11), Hfb(4,11), tf, alfa, Y, a,    *
*          alpha, beta                                               *
*                                                                    *
*  Called by : Main program                                          *
*                                                                    *
*  Purpose : Calculates the maximum temperature and corresponding    *
*            probability of damage for the burst missile configur-   *
*            ation defined by cells Bk                               *
*                                                                    *
*  Calls : ClVar - Cell Variables                                    *
*          MsChar - Missile characteristics (IF NECESSARY)           *
*                                                                    *
**********************************************************************

      SUBROUTINE TCalc(B1,B2,B3,B4,Mn,tmax,tb1,maxb,Ta0,rhocep,
     +                 theta,sumPd,LowT,HiT,lt)

      INTEGER B1,B2,B3,B4,Mn,tb1,nb,cellno,tb,ab
      REAL tmax,MaxT,a,tf,Y,alfa,dQ,T1,T2,SR,CF,Tau,Ta0,sumPd,Pdi,Z
      REAL ax,ay,rhocep(10),theta(10),vel,alt,drd,ang,h,Temp,dCT,Zp,Pz
      REAL Hfb,lt,v,height,x,phi,t
      REAL LowT,HiT
      LOGICAL newbst
      COMMON /block2/ax(100),ay(100)
      COMMON /block3/vel(11),alt(11),drd(11),ang(11),h(11),Temp(11)
      COMMON /block4/dCT(4,11),Hfb(4,11)
      COMMON /block5/tf,alfa,Y,a,alpha,beta

      nb = 1
      j = 0
      T1 = Ta0
      T2 = T1 + 1.0
C     Condition for continuing calculation :
   10 IF (T2 .GE. T1 .AND. j .LE. 9) THEN
         j = j + 1
C        If new burst has detonated during j, increase # of bursts by 1
         IF (nb .LT. maxb .AND. j*tmax .GE. 2*nb) THEN
            nb = nb + 1
            newbst = .TRUE.
         ELSE
            newbst = .FALSE.
         END IF
         dQ = 0.0
C        Calculate dQ for each burst k at time step j
         DO 20 k = 1,nb
```

```
            tb = tb1 + 2*(k-1)
            sb = (tb+2)/2
            IF (k .EQ. 1) THEN
               cellno = B1
            ELSE IF (k .EQ. 2) THEN
               cellno = B2
            ELSE IF (k .EQ. 3) THEN
               cellno = B3
            ELSE IF (k .EQ. 4) THEN
               cellno = B4
            END IF
C           If new burst has just occurred, must calculate missile
C           characteristics at midpoint between tbk and j*tmax
            IF (newbst .AND. k .EQ. nb) THEN
               t = tb1-lt + j*tmax - (tb1+j*tmax-tb)/2.
               CALL MsChar(t,v,height,x,phi,ax(Mn))
               CALL ClVar(SR,CF,Tau,v,height,x,phi,ax(nb),ay(sb),
     +               ay(Mn),rhocep(cellno),theta(cellno),Hfb(k,j))
            ELSE
               CALL ClVar(SR,CF,Tau,vel(j),alt(j),drd(j),ang(j),ax(sb)
     +                 ,ay(sb),ay(Mn),rhocep(cellno),theta(cellno),
     +                  Hfb(k,j))
            END IF
            dQ = dQ+dCT(k,j)*tf*Y*Tau*CF*4.186e12/(4.*3.14159*(SR**2))
   20     CONTINUE
          T2 = (T1*(a-h(j)*tmax/2.)+h(j)*tmax*Temp(j)+alfa*dQ)/
     +          (a+h(j)*tmax/2.)
          IF (T2 .GT. T1) T1 = T2
          GOTO 10
        END IF
        MaxT = T1
        IF (MaxT .LT. LowT) LowT = MaxT
        IF (MaxT .GT. HiT) HiT = MaxT
C       Calculate the probability of damage and add to running sum
        Z = (ALOG(MaxT) - alpha)/beta
        Zp = ABS(Z)
        Pz = 1.-1./(2.*(1.+.196854*Zp+.115194*(Zp**2)+.000344*
     +              (Zp**3)+.019527*(Zp**4))**4)

        IF (Z .GE. 0.0) THEN
           Pdi = Pz
        ELSE
           Pdi = 1.0 - Pz
        END IF
        sumPd = sumPd + Pdi

        END
```

```
*********************************************************************
*                                                                   *
*                 SUBROUTINE ClVar  - Cell Variables                *
*                                                                   *
*  Passed : SR, CF, Tau, vel(j), alt(j), drd(j), ang(j), sx(sb),    *
*           sy(sb), sy(Mn), rhocep(cellno), theta(cellno), Hfb(k,j) *
*                                                                   *
*  Called by : Subroutine TCalc                                     *
*                                                                   *
*  Purpose : Calculates SR, CF, and Tau using the given missile     *
*            and burst characteristics                              *
*                                                                   *
*********************************************************************

      SUBROUTINE ClVar(SR,CF,Tau,v,z,x,phi,dgzx,dgzy,Mny,rocepi,
     +                 thetai,fbh)

      REAL SR,CF,Tau,v,z,x,phi,dgzx,dgzy,Mny,rocepi,thetai,fbh
      REAL rhoi,xb,yb,gr,cosphi

C  Note: CEP = 200 meters
      rhoi = rocepi*200
      xb = dgzx + rhoi*COS(thetai)
      yb = dgzy + rhoi*SIN(thetai)
      gr = SQRT((Mny-yb)**2 + (x-xb)**2)
      SR = SQRT(gr**2 + (z-fbh)**2)
      IF (v .EQ. 0) THEN
         CF = 1.0
      ELSE
         cosphi = ((x-xb)*v*COS(phi)+(z-fbh)*v*SIN(phi))/(SR*v)
         CF = SQRT(1. - cosphi**2)
      END IF
      Tau=EXP(-.02455-6.439e-5*SR-1.407e-9*(SR**2)+1.792e-14*(SR**3))
      END
```

```
*****************************************************************
*                                                               *
*         SUBROUTINE InitCh - Initial Characteristics           *
*                                                               *
*  Passed : tb1, lt, tmax, Mn, TaO                              *
*  COMMON : sx(100), sy(100), vel(11), alt(11), drd(11), ang(11),*
*           h(11), Temp(11)                                     *
*                                                               *
*  Called by : Main program                                     *
*                                                               *
*  Purpose : Calculates and stores missile characteristics, h, and *
*            ambient air temperature for each time step up to j = 11. *
*            Values are calculated at the time step midpoint t = *
*            (tb1-lt) + (j-.5)*tmax                             *
*                                                               *
*  Calls : MsChar - Missile characteristics                     *
*          HCalc - Heat transfer coefficient calculation        *
*                                                               *
*****************************************************************

      SUBROUTINE InitCh(tb1,lt,tmax,Mn,TaO)

      INTEGER tb1,Mn
      REAL lt,tmax,t0,tm,v0,z0,x0,phi0,h0,TaO
      REAL vel,alt,drd,ang,h,Temp,sx,sy
      COMMON /block2/sx(100),sy(100)
C Only sx will be used in this subroutine
      COMMON /block3/vel(11),alt(11),drd(11),ang(11),h(11),Temp(11)

C Find air temperature at t0
      t0 = tb1 - lt
      CALL MsChar(t0,v0,z0,x0,phi0,sx(Mn))
      CALL HCalc(v0,z0,h0,TaO)
C Calculate missile characteristics, h, and Temp for midpoint of each
C  time step j
      DO 10 j = 1,11
         tm = t0 + (j-.5)*tmax
         CALL MsChar(tm,vel(j),alt(j),drd(j),ang(j),sx(Mn))
         CALL HCalc(vel(j),alt(j),h(j),Temp(j))
   10 CONTINUE
      END
```

```
************************************************************************
*                                                                      *
*          SUBROUTINE  FBCalc - Fireball Calculations                  *
*                                                                      *
*  Passed : maxb, tb1, tmax, rise, W                                   *
*  COMMON : dCT(6,11), Hfb(6,11)                                       *
*                                                                      *
*  Called by : Main Program                                            *
*                                                                      *
*  Purpose : Calculates dCT and Hfb for each burst k at each time      *
*            step j                                                    *
*                                                                      *
*  Calls : Function CT - calculates CT at time tp                      *
*                                                                      *
************************************************************************

      SUBROUTINE FBCalc(maxb,tb1,tmax,rise,W)

      INTEGER maxb,tb1,nb,tb,rise
      REAL tmax,CT,dCT,Hfb,CTu,CTd,W,t
      LOGICAL newbst
      COMMON /block4/dCT(4,11),Hfb(4,11)

C Initialize arrays
      DO 5 k = 1,4
        DO 5 j = 1,11
          dCT(k,j) = 0.0
          Hfb(k,j) = 0.0
    5 CONTINUE
      nb = 1
C For each time step j:
      DO 10 j = 1,11
C         If a new burst has detonated during j, increase # of bursts
         IF (nb .LT. maxb .AND. j*tmax .GE. 2*nb) THEN
             nb=nb+1
             newbst = .TRUE.
         ELSE
             newbst = .FALSE.
         END IF
C         For each burst k that has occured :
         DO 20 k = 1,nb
            tb = tb1 + 2*(k-1)
C            Find dCT for burst k at time tp = t/tmax
            tp = (tb1 + j*tmax - tb)/tmax
               IF (tp .LE. 10) THEN
                  CTu = CT(tp)
                  tp = tp - 1.0
                  IF (tp .GT. 0) THEN
                     CTd = CT(tp)
                  ELSE
                     CTd = 0.0
                  END IF
```

```
            dCT(k,j) = CTu - CTd
         END IF
C        If required, find fireball height for burst k at time t
         IF (rise .EQ. 1) THEN
C           If a new burst has just occurred during j, then
C           Hfb is calculated for the midpoint of the time
C           from tbk to t = j*tmax.  Otherwise, Hfb is
C           calculated at the midpoint of time step j
            IF (newbst .AND. k .EQ. nb) THEN
               t = (tb1 + j*tmax - tb)/2.
            ELSE
               t = tb1 + (j-.5)*tmax - tb
            END IF
            Hfb(k,j) = 21640.6*(W**.177)*(1.-(1.-t/240.)**2)
         END IF
   20 CONTINUE
   10 CONTINUE

      END
```

```
***********************************************************************
*                                                                     *
*                            FUNCTION CT                              *
*                                                                     *
*  Passed : CT, tp                                                    *
*                                                                     *
*  Called by : Subroutine FBCalc - Fireball calculations              *
*                                                                     *
*  Purpose : Calculates CT at normalized time tp from equations       *
*            determined using linear regression on Figure 7.84 of     *
*            Glasstone and Dolan (Glasstone and Dolan, 1977:311)      *
*                                                                     *
***********************************************************************

      FUNCTION CT (tp)

      IF (tp .LE. .75) THEN
         CT = -.02*tp + .24*(tp**2)
      ELSE IF (tp .GT. .75 .AND. tp .LE. 1.5) THEN
         CT = .32*tp - .12
      ELSE IF (tp .GT. 1.5 .AND. tp .LE. 2.5) THEN
         CT = -.257219+.558415*tp-.0969039*(tp**2)
      ELSE IF (tp .GT. 2.5 .AND. tp .LT. 10.0) THEN
         CT = .335808+.0949904*tp-.00494514*(tp**2)
      ELSE IF (tp .EQ. 10.0) THEN
         CT = .80
      END IF
      END
```

```
*************************************************************************
*                                                                       *
*              SUBROUTINE MsChar - Missile Characteristics              *
*                                                                       *
*  Passed : t, vel(j), alt(j), drd(j), ang(j), ax(Mn)                   *
*  COMMON : vdata(51), zdata(51), xdata(51), angdat(51)                 *
*                                                                       *
*  Called by : Subroutine InitCh - Initialize characteristics           *
*                                                                       *
*  Purpose : Calculates missile velocity, altitude, down-range-         *
*            distance, and flight path angle at time t using data       *
*            files created from plot of missile behavior (figure 1)     *
*                                                                       *
*************************************************************************

      SUBROUTINE MsChar(t,v,z,x,phi,Mnx)

      REAL t,v,z,x,phi,Mnx
      INTEGER td,tu
      REAL vdata,zdata,xdata,angdat
      COMMON /block1/vdata(51),zdata(51),xdata(51),angdat(71)

C Determine lower and upper limit of interpolation
      td = INT(t)
      tu = td + 1
C Find velocity, altitude, and distance from silo
      If (t .GT. 50) THEN
         v = 105.*t - 1350.
         z = 2480.*t - 65000.
         x = 3700.*t - 127000.

      ELSE
C        Use linear interpolation to calculate value
         v = (t-td)*(vdata(tu+1)-vdata(td+1)) + vdata(td+1)
         z = (t-td)*(zdata(tu+1)-zdata(td+1)) + zdata(td+1)
         x = (t-td)*(xdata(tu+1)-xdata(td+1)) + xdata(td+1)
      END IF
C Find flight path angle
      IF (t .GT. 70) THEN
         phi = 30.8 - .1*(t-70)
      ELSE
         phi=(t-td)*(angdat(tu+1)-angdat(td+1))+angdat(td+1)
      END IF
C Convert to meters and radians and calculate down-range-distance from x=0
      v = v*.3048
      z = z*.3048
      x = x*.3048 + Mnx
      phi = phi*3.14159/180.
      END
```

```
**********************************************************************
*                                                                    *
*     SUBROUTINE HCalc - Heat Transfer Coefficient Calculation       *
*                                                                    *
*  Passed : vel(j), alt(j), h(j), Temp(j)                            *
*                                                                    *
*  Called by : Subroutine InitCh - Initialize characteristics        *
*                                                                    *
*  Purpose : Calculates h using missile velocity and ambient air     *
*            properties at missile altitude.  Air properties are     *
*            found using equations from US Standard Atmosphere       *
*            (NOAA, 1976:6-30)                                       *
*                                                                    *
**********************************************************************

      SUBROUTINE HCalc(v,z,htc,Ta)

      REAL v,z,htc,Ta
      REAL cp,Re,Pr,Nu,xm,P,rhoa,mu,kappa

C  Find ambient air conditions at z using US Standard Atmosphere equations
      IF (z .LT. 11000.) THEN
         Ta = 288.15 - .006545*z
         P = 101300.*((288.15/Ta)**(-.034164/.006545))
      ELSE IF (z .GE. 11000. .AND. z .LT. 20000.) THEN
         Ta = 216.65
         P = 22680.*EXP(-.034164*(z-11000.)/216.65)
      ELSE IF (z .GE. 20000. .AND. z .LT. 32000.) THEN
         Ta = 216.65 + .001*(z-20000.)
         P = 5528.*((216.65/Ta)**(.034164/.001))
      ELSE IF (z .GE. 32000. .AND. z .LT. 47000.) THEN
         Ta = 228.65 + .0028*(z-32000.)
         P = 888.8*((228.65/Ta)**(.034164/.0028))
      ELSE
         WRITE(*,*)'z > 47000'
      END IF
      rhoa = .003484*P/Ta
      mu = 1.458e-6*(Ta**1.5)/(Ta + 110.4)
      kappa = 2.64638e-3*(Ta**1.5)/(Ta + 245.4*(10**(-12./Ta)))

C Calculate the heat transfer coefficient at point xm
      cp = 240.*4.184
      xm = 5.5
      Re = rhoa*v*xm/mu
      Pr = mu*cp/kappa
      IF (Re .LE. 500000.) THEN
         Nu = .332*(Pr**(1./3.))*(Re**.5)
      ELSE
         Nu = .0296*(Pr**(1./3.))*(Re**(4./5.))
      END IF
      htc = Nu*kappa/xm
      END
```

```
************************************************************************
*                                                                      *
*                  SUBROUTINE GData - Get Data                         *
*                                                                      *
*  COMMON : vdata(51), zdata(51), xdata(51), angdat(51), sx(100),      *
*           sy(100)                                                    *
*                                                                      *
*  Called by : Main program                                            *
*                                                                      *
*  Purpose : Initializes missile characteristic arrays by reading      *
*            data from external files and initializes silo position    *
*            arrays                                                    *
*                                                                      *
************************************************************************

      SUBROUTINE Gdata

      REAL vdata,zdata,xdata,angdat,sx,sy,dx,dy,rhocep,theta
      COMMON /block1/vdata(51),zdata(51),xdata(51),angdat(71)
      COMMON /block2/sx(100),sy(100)

C Read in data from files
   10 FORMAT(BN,F8.1)
   20 FORMAT(BN,F8.2)
      OPEN (8,file='veldata.txt',status='old',
     +      access='sequential',form='formatted')
      REWIND 8
      DO 30 i = 1,51
         READ(8,10)vdata(i)
   30 CONTINUE
      CLOSE (8)
      OPEN (9,file='altdata.txt',status='old',
     +      access='sequential',form='formatted')
      REWIND 9
      DO 40 i = 1,51
         READ(9,10)zdata(i)
   40 CONTINUE
      CLOSE (9)
      OPEN (10,file='drddata.txt',status='old',
     +      access='sequential',form='formatted')
      REWIND 10
      DO 50 i = 1,51
         READ(10,10)xdata(i)
   50 CONTINUE
      CLOSE (10)
      OPEN (11,file='degdata.txt',status='old',
     +      access='sequential',form='formatted')
      REWIND 11
      DO 60 i = 1,71
         READ(11,20)angdat(i)
   60 CONTINUE
      CLOSE (11)
```

```
C Calculate and store silo x and y position
      dx = 519.62
      dy = 300.
      DO 70 i = 0,95,5
        sy(i+1) = 0.0
        sy(i+2) = 2.*dy
        sy(i+3) = 4.*dy
        sy(i+4) = dy
        sy(i+5) = 3.*dy
   70 CONTINUE
      j = 0
      DO 80 i = 0,38,2
        sx(j+1) = i*dx
        sx(j+2) = i*dx
        sx(j+3) = i*dx
        sx(j+4) = (i+1)*dx
        sx(j+5) = (i+1)*dx
        j = j + 5
   80 CONTINUE
      END
```

```
10 REM ********  THERMAL INTERACTION / Non-Cumulative Effect  *********
20 DIM VDATA(51),ZDATA(51),XDATA(51),ANGDATA(71),MLT(100),SX(100),SY(100)
30 GOSUB 1100 : REM Initialize vel,alt,drd,ang,mlt,sx,and sy arrays
40 PI = 3.14159 : DIM CT(11)
50 DATA 0.0,0.2,0.488,0.5763,0.6386,0.6871,0.7277,0.7584,0.7792,0.7902,0.80
60 FOR I = 0 TO 10 : READ CT(I) : NEXT I
70 DIM RHOCEP(10),THETA(10)
80 GOSUB 1020 : REM Call subroutine to initialize N cell centroid co-ords
90 INPUT"Yield (KT)";Y : INPUT "N,maxb,rise";N,MAXB,RISE
100 HOB = 0 : ISS = 619 : ISK = 809 : CEP= 200
110 ALPHA = (1!/2!)*LOG(ISS*ISK): BETA = (1!/4.108)*LOG(ISK/ISS)
120 TMAX = .0417*Y^.44
130 RHO=2700 : ALFA=.5 : CV=900 : D =.001 :REM values for Al
140 A = CV*RHO*D : C = ALFA*PMAX/(A*4!*PI) : TF = .18 : XM = 5.5
150 INPUT"Silo # and designated launch time";SN,LT
160 INPUT"Time of first burst (must be >= launch time and even)";TB1
170 LPRINT:LPRINT"Missile #";SN;" Launch t:";LT;" First burst at time :";TB1
180 LPRINT"Y(KT):";Y,"N:";N,"maxb:";MAXB,"rise:";RISE
185 LOWT = 1000000! : HIT = 0!
190 GOSUB 230 : REM Find cumulative Ps
200 GOTO 90
210 END
```

```
220 REM ************************************************************************
230 REM Subroutine to calculate cum Ps for first four bursts affecting SN
240 FOR NB = 1 TO MAXB
250 TB = TB1+2*(NB-1) : SB = (TB+2)/2
260 DGZX = SX(SB) : DGZY = SY(SB) : SUMPD = 0!
270 TO=TB - LT
280 FOR I = 1 TO N
290 PRINT NB,I
300     J = 0 : T = TO : GOSUB 510 : Z = ALT : GOSUB 880 : T1 = TA
310     REM
320         J = J + 1 : T = TO + (J-.5)*TMAX
325         IF RISE = 1 THEN HFB = 21840.8*(Y/1000!)^.177*(1!-(1!-T/240!)^2)
330         GOSUB 510 : GOSUB 880 : GOSUB 780 : TEMP = TA
335         DCT = CT(J)-CT(J-1)
340         DQ = DCT*TF*Y*TAU*CF*4.186E+12/(4*PI*SR^2)
350         T2 = (T1*(A -H*TMAX/2)+H*TMAX*TEMP+ALFA*DQ)/(A+H*TMAX/2)
360         LPRINT USING"## ";J;: LPRINT USING"#####.##  ";T2
370         REM IF T1 > T2 THEN GOTO 395
380         T1 = T2 : IF J < 10 THEN GOTO 320
390     REM
395     IF T1 < LOWT THEN LOWT = T1 : IF T1 > HIT THEN HIT = T1
400     B = (LOG(T1)-ALPHA)/BETA
410     BP = ABS(B)
420     PB = 1-1!/(2!*(1!+.196854*BP+.115194*BP^2+.000344*BP^3+.019527*BP^4)^4)
430     IF B>= 0 THEN PDI = PB ELSE PDI = 1-PB
440     SUMPD = SUMPD + PDI
450 NEXT I
460 PD=(1!/N)*SUMPD : PS=1!-PD  : IF NB=1 THEN CUMPS=PS ELSE CUMPS = CUMPS*PS
460 NEXT NB
482 REM LPRINT"Low T = ";L
485 REM LPRINT"Cumulative Ps for ";:LPRINT USING"## ";MAXB;:LPRINT"bursts is:
";CUMPS
490 RETURN
```

```
500 REM ************************************************************************
510 REM Subroutine to find missile vel,alt,drd, and ang at time T
520 TD = FIX(T) : TU = TD + 1
530 IF T > 50 THEN GOTO 610
540    IF T=TD THEN VL=VDATA(T):AL=ZDATA(T):DR=XDATA(T):GOTO 620
550      REM
560         VL = ((T-TD)/(TU-TD))*(VDATA(TU)-VDATA(TD)) + VDATA(TD)
570         AL = ((T-TD)/(TU-TD))*(ZDATA(TU)-ZDATA(TD)) + ZDATA(TD)
580         DR = ((T-TD)/(TU-TD))*(XDATA(TU)-XDATA(TD)) + XDATA(TD)
590         GOTO 620
600      REM
610         VL = 105*T-1350 : AL = 2480*T - 65000! : DR=3700*T-127000!
620    IF T>70 THEN ANG= 30.6 - .1*(T-70) : GOTO 650
630    IF T = TD THEN  ANG = ANGDATA(T) : GOTO 650
640      ANG=(((T-TD)/(TU-TD))*(ANGDATA(TU)-ANGDATA(TD))+ANGDATA(TD))
650 VEL = VL*.3048:ALT = AL*.3048:DRD = DR*.3048+SX(SN): ANG = ANG*2!*PI/360!
660 RETURN



670 REM ************************************************************************
680 REM Subroutine to calculate SR, CF, and TAU given ALT
690      DELTAZ = ALT-HFB : RHOI = RHOCEP(I)*CEP
700      XB = DGZX + RHOI*COS(THETA(I)) : YB = DGZY + RHOI*SIN(THETA(I))
710      GR = SQR((SY(SN)-YB)^2 + (DRD-XB)^2)
720      SR = SQR(GR^2 + DELTAZ^2) : SRX = DRD-XB : SRZ = DELTAZ
730      IF VEL = 0 THEN CF = 1 : GOTO 760
740      COSPHI = (SRX*VEL*COS(ANG)+SRZ*VEL*SIN(ANG))/(SR*VEL)
750      CF = SQR(1!-COSPHI^2)
760      TAU=EXP(-.02455-6.439E-05*SR-1.407E-08*SR^2+1.792E-14*SR^3)
770 RETURN



780 REM ************************************************************************
790 REM Subroutine to calculate heat transfer coefficient, h (J/m2-s-K)
800      Z = ALT : GOSUB 880 : REM Find ambient air rhoa,TA,mu,ka at z
810      CP = 240*4.184
820      RE = RHOA*VEL*XM/MU
830      PR = MU*CP/KA
840  IF RE<=500000! THEN NU=.332*PR^(1/3)*RE^.5 ELSE NU=.0296*PR^(1/3)*RE^(4/5)
850 H= NU*KA/XM
860 RETURN
```

```
870 REM ******************************************************************
880 REM Subroutine : US Standard Atmosphere to 47 km
890 IF Z < 11000 THEN LK = -.006545 : PK = 101300! : TK = 288.15 : ZK = 0
900 IF Z>=11000 AND Z<20000 THEN LK=0 : PK=22690! : TK=216.65 : ZK=11000
910 IF Z>=20000 AND Z<32000 THEN LK=.001 : PK=5528 : TK=216.65 : ZK=20000
920 IF Z>=32000 AND Z<47000! THEN LK=.0028 :PK=868.8 : TK=228.65 :ZK=32000
930 IF Z >= 47000! THEN PRINT "z >= 47000, so consult NOAA for values"
940 IF LK = 0 THEN GOTO 950 ELSE GOTO 960
950      P = PK*EXP(-.034164*(Z - ZK)/TK) : TA = TK : GOTO 970
960      TA = TK + LK*(Z - ZK) : P = PK*(TK/TA)^(.034164/LK) : GOTO 970
970 RHOA = .003484*P/TA
980 MU = 1.458E-06*TA^1.5/(TA + 110.4) : REM (kg/m-s)
990 KA = 2.64638E-03*TA^(1.5)/(TA + 245.4*10^(-12!/TA)) : REM J/(m-s-K)
1000 RETURN


1010 REM ******************************************************************
1020 REM Subroutine to initialize coordinates of N equal prob cell grid
1030 RHOCEP(1)=0! : FOR I=2 TO 5 : RHOCEP(I)=.7109 : NEXT I
1040 FOR I = 6 TO 10 : RHOCEP(I) = 1.509 : NEXT I
1050 THETA(1)=0! : THETA(2)=(2*PI/4)/2 : THETA(6) = 0!
1060 FOR I = 3 TO 5 : THETA(I) = THETA(I-1)+(2*PI/4) : NEXT I
1070 FOR I = 7 TO 10 : THETA(I) = THETA(I-1)+(2*PI/5) :NEXT I
1080 RETURN


1090 REM ******************************************************************
1100 REM Subroutine to initialize vel,alt,drd,ang,mlt,sx,and sy arrays
1110 OPEN "I",#1,"B:veldata.txt"
1120 FOR I = 0 TO 50 : INPUT #1,VDATA(I) : NEXT I : CLOSE #1
1130 OPEN "I",#2,"B:altdata.txt"
1140 FOR I = 0 TO 50 : INPUT #2,ZDATA(I) : NEXT I : CLOSE #2
1150 OPEN "I",#3,"B:drddata.txt"
1160 FOR I = 0 TO 50 : INPUT #3,XDATA(I) : NEXT I : CLOSE #3
1170 OPEN "I",#1,"B:degdata.txt"
1180 FOR I = 0 TO 70 : INPUT #1,ANGDATA(I) : NEXT I : CLOSE #1
1190 OPEN "I",#1,"B:MLO.txt"
1200 FOR I = 1 TO 100 : INPUT #1,MLT(I) : NEXT I : CLOSE #1
1210 DX = 519.62 : DY = 300
1220 FOR I = 0 TO 95 STEP 5
1230    SY(I+1)=0 : SY(I+2)=2*DY : SY(I+3)=4*DY : SY(I+4)=DY : SY(I+5)=3*DY
1240 NEXT I
1250 J = 0
1260 FOR I = 0 TO 38 STEP 2
1270  SX(J+1)=I*DX:SX(J+2)=I*DX:SX(J+3)=I*DX:SX(J+4)=(I+1)*DX:SX(J+5)=(I+1)*DX
1280  J = J+5
1290 NEXT I
1300 RETURN
```

Appendix L. Results of Dust Shielding Analysis

The following table presents data which shows that the geometric treatment of dust shielding does not affect the transmittance to any missiles of interest (i.e., missiles past #31). The values of ϕ given are the maximum values reached during a four-burst, 1-cell CEP scenario.

TABLE L-I

RESULTS OF DUST SHIELDING ANALYSIS

Missile launch time: 0 sec
Time of first burst: 0 sec
1-cell CEP
Fireball rise considered

$\phi > 20°: \tau = 0$

Missile #	Maximum ϕ (°) for Burst #			
	1	2	3	4
11	27.4	18.9	9.5	1.5
14	22.9	16.1	8.4	1.2
15	22.1	16.1	8.7	1.1
16	19.9	13.6	7.0	1.0
21	15.5	10.6	5.5	0.7
31	10.7	7.3	3.9	0.5
41	8.2	5.6	2.9	0.3
51	6.8	4.5	2.4	0.3

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In current survivability studies, the nuclear bursts are treated as independent events. Using this method, the effect of thermal radiation from one fireball at a time is studied. This treatment does not consider the cumulative effects of receiving thermal radiation from more than one fireball at a time. The purpose of this thesis was to develop a computer program to model the cumulative effects of thermal radiation and compare these results to those from the noncumulative case.

The scenario studied was the Peacekeeper Dense Pack missile system. The missile field was subjected to a walk attack of 2 MT weapons every two seconds. The aiming error of the incoming RV was modeled using a 10-cell circular error probable (CEP) area around the designated ground zero, and the probability of damage due to an RV was calculated using a cumulative log-normal distribution function.

In order to model the temperature rise of the missile skin, an energy balance was made over a unit area of skin surface and then solved using the thin skin approximation and finite differences. The resulting equation gave an expression for the skin temperature at a time t after the first burst detonated. The maximum temperature reached was then used to calculate the probability of damage to the missile skin.

To model cumulative thermal effects, the amount of thermal radiation emitted from each burst was added together to calculate the skin temperature. This method resulted in temperatures that were significantly higher than the temperatures calculated for each burst independently. Consequently, cumulative thermal effects proved to have a greater region of no survival than noncumulative thermal effects and also blast effects.